Advanced Turbulence Theory Exam

Instructor: William K. George, 2013-04-4

Instructions

- You can feel free to discuss this among yourselves and even work together. Or you can work alone. In fact I would prefer that you shared your knowledge, so all of you learn everything. This is not a competition. You may ask for help as well, but don't be surprised if the people you ask don't know the answers, since some of this will seem new to them as well. And you, of course, can ask me questions or email me to ask about the notes or the lectures (e.g., mistakes in them, etc.), and of course to clarify a question on the exam.
- You should, however, turn in your own paper, **prepared entirely by you**, stating clearly for each problem with whom you worked and what their contribution was. Please don't insult me (or embarrass yourselves) by just copying each other. That will not make me happy, and I will probably ask you for an oral exam if I think you have done this (or just fail you outright).
- If you use a computer (matlab, excel, etc.) append a copy of your script for each problem, clearly labeling what the program was used for.
- Please do ALL problems, even it takes a bit longer. I've tried to in one form or another make you use most of the course material. Some will be easier on a spreadsheet, others with matlab, all with using your brain.
- If you use any references, please cite them. Recognize that I will probably know immediately if you have taken something from a reference instead of thinking about it or using your knowledge, since much of what the authors say about their own work are things I do not believe are correct — nor I suspect will you. :-) Also since one of the objectives of this course was to learn to read the literature yourself, I've provided you with a number of papers and a presentation by me and my co-workers to get you started. :-)
- Please hand this *in paper form* in **at the latest by 15 April 2012**.
- This will count 100% of your grade.
- You've been a great class, and I've tried hard to think of way to let you prove that to yourselves. **Good luck on the exam!**

Part I: Using your DNS

1. Derive the isotropic integral and derivative relations between the three-dimensional energy spectrum function, $E(k)$, and the one-dimensional spectra, $F^{(1)}_{1,1}(k_1)$ and $F^{(1)}_{2,2}(k_1)$.

2. You have carried out simulations of forced turbulence using periodic boundary conditions. In a presentation in 2012 at Imperial (George,W.K., Malloupas, G and van Wachem, B. 2nd Int. UK-Japan Symposium on Fractal Turbulence), it was argued that averaging in time was very different than averaging in space. In particular, the time correlation functions and spectra computed (properly, meaning not making it artificially periodic) transformed into each other with the proper window functions. But the same was not true for the spatial correlations and spectra for which window functions were clearly not needed. Discuss how the main points of this presentation relate to the forced turbulence simulations you have done for J-P Laval's class.

3. Use your previous 64 x 64 simulation for **decaying turbulence** or make a new one. Then:

   - (i) Compute the one-dimensional wavenumber spectra at various times during the decay, making sure you are beyond the initial transient and onto something resembling a power law decay.
   - (ii) Now compute the 3-dim. energy spectrum function, either directly or from your 1-dim spectra. Why can you do the latter?
   - (iii) How can you tell you are getting on this asymptotic behavior without being fooled by the arbitrary choice of a virtual origin or log-log plots? (Hint: look at the papers provided, especially the ones about isotropic decay.)
   - (iv) Wang and George(2003) suggest that it is important to think about where the peak in the 3-dim.energy spectrum function is relative to the lowest wavenumber. How do your data measure up to their criterion?
   - (v) Scale your spectra using the Taylor microscale and the turbulence intensity to see if the George 92 (Phys Fluids 1992) theory is consistent with them. Look at the comments of Wang and George (JFM 2003) and also Wang et al.2000 (unpublished) and George et al. from the Australasian Fluids meeting in 2001 or the last part of the 2012 ASME paper. How much do you think your results might suffer from the issues raised in these papers, especially box-size and resolution?

4. Review carefully the consequences of the assumptions of local homogeneity, local axisymmetry, and local isotropy on the mean square derivative moments. Summarize them here.

Part II Using real PIV data

5. You will be provided with a set of velocity data by Jean-Marc Foucaut from his PIV measurements. They were measured in the boundary layer of the big wind tunnel at Lille using PIV in two different planes. These data have not been previously analyzed, so you will be the first. Your mission (Mr Phelps, should you choose to accept it – from Mission Impossible :-)) is to test whether and to what extend the above relations can be used to describe them

   - (i) First use the local homogeniety assumption, together with the continuity equation, to see whether you can decide which measurements might be better than others. For example, any squared quantity (like $\langle (\partial u/\partial x)^2 \rangle$) will be contaminated more by noise than unsquared quantities like $\langle \partial u/\partial y \; \partial v/\partial x \rangle$ since the noise is likely to be less than perfectly correlated. So if you can compute squared quantities from those that are not, they might be better. A clue will be that they are less, especially if you can show consistency in the amount of noise.
   - (ii) Now test the basic deductions of local axisymmetry and local isotropy. Which, if any, of the these relations are satisfied?
   - (iii) Now compute the dissipation using the various estimates you can construct. Which do you think is the closest to the actual value?
   - (iv) Now compute the Kolmogov microscale, η_{Kol}, using what you believe to be your best estimate. Typically $2\pi\eta_{Kol}$ is interpreted to be the physical distance associated with the Kolmogorov microscale (since $k\eta_{Kol} = 1$ is the wavenumber below which 99 % of the dissipation occurs). So to measure 99 % of the dissipation you really need to be able to resolve about half of this; i.e., $\pi\eta_{Kol}$. How small would the PIV volume have to be to completely resolve the dissipation?
   - (v) Finally, pick a reference point and compute the $\langle u(x, y, z, t)u(x + r, y, z, t) \rangle$ and $\langle v(x, y, z, t)v(x + r, y, z, t) \rangle$ correlations. Now fit parabolas near $r = 0$ and detemine the corresponding Taylor microscales, and from them compute the mean velocity gradients $\langle (\partial u/\partial x)^2 \rangle$ and $\langle (\partial v/\partial x)^2 \rangle$. How do these compare to your results obtained directly from the derivative moments?
   - (vi) Assuming that noise is mostly going to be an issue at exactly $r = 0$, how can you use your knowledge of turbulence and your parabolic curve fit to get an improved estimate for $\langle u^2 \rangle$ and $\langle v^2 \rangle$ over what you would get by simply squaring and averaging the velocity data?