Examination of LSE-based velocity field estimations using instantaneous, full field measurements in the annular mixing layer.

Daniel Ewing
and
Joseph H. Citriniti
Thermo- and Fluid Dynamics
Chalmers University of Technology
S-412 96 Gothenburg, Sweden

Submitted to 11th Symposium on Turbulent Shear Flows
November 13, 1996

It is well recognized that coherent structures play an important role in the near field of turbulent free-shear layers, such as the axisymmetric shear layer. There are numerous different technics that be used to identify the coherent structures (*cf* Bonnet and Glauser, 1994); however, only a few of these can be used to study the dynamics of the structures.

One such technic, is the Proper Orthogonal Decomposition (POD) introduced by Lumley (1967). In this technic, the coherent structures are identified as those that yield the largest mean square projection onto the velocity field. This maximization process yields the integral eigenvalue problem

$$\int R_{i,j}(\tilde{x},\tilde{x}')\Phi_j^n(\tilde{x}')d\tilde{x}' = \lambda^n \Phi_j^n(\tilde{x}), \tag{1}$$

so the structures are identified using the two-point velocity correlation, $R_{i,j}(\tilde{x},\tilde{x}')$. Here, $\Phi_j^n(\tilde{x})$ is an orthogonal basis that can be used to reconstruct the field and λ^n is the contribution of each of the modes to the average turbulent kinetic energy. Since the two-point correlation is an averaged quantity, it is straightfoward to obtain enough information to accurately estimate the integral in equation 1 and identify the eigenfunctions for the flow.

In order to understand how these structures contribute to the dynamics of the flow however, it is necessary to determine the instantaneous coefficients of the functions; *i.e.*,

$$a^n(t) = \int u_j(\tilde{x}')\Phi_i^{*n}(\tilde{x}')d\tilde{x}'. \tag{2}$$

A low-order model of the layer can then be constructed using only the contribution of the most energetic modes; *e.g.*,

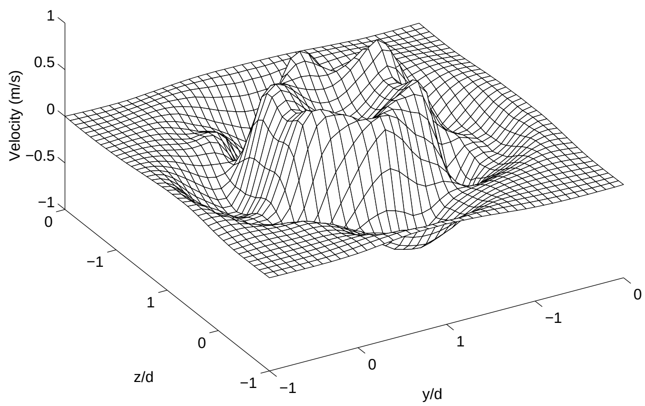
$$u_j^N(\tilde{x}) = \sum_{n=1}^{N} a^n(t)\Phi_j^n(\tilde{x}), \tag{3}$$

where N is a user specified number. This technic is particularly useful for studying the dynamics of the large-scale structures in high-Reynolds-number flow because it removes the influence of the incoherent small-scale motions that often obscure the larger motions. For example, Glauser *et al* (1987) found that the velocity traces in an axisymmetric shear layer could be well produced using only three modes, *i.e.,* $N = 3$, when the POD was applied to velocities measured in the radial direction of this flow. In addition, it was later demonstrated that the dynamics of the ring vorticies in the annular shear layer could be well represented using only a single mode (*e.g.*, Ukeiley *et al.*, 1994).

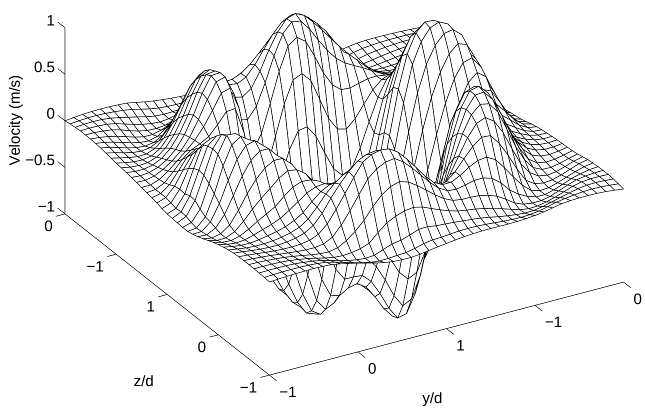
In a later application of the POD to an entire $r - \theta$ plane in the annular shear layer (at a distance of $x/D = 3$ downstream of the nozzle), Glauser (1989) found that there was a significant amount of energy in the azimuthal Fourier modes with mode numbers 4, 5, and 6. Corke *et al.* (1994) later found that the entrainment in the axisymmetric shear layer could be enhanced quicker when these modes were forced then when the 0 or ring mode was forced. The exact influence of these modes in the dynamics of the shear layer was not known however, until a recent study by Citriniti (1996). He was able to determine the instantaneous coefficients for the POD modes in the $r - \theta$ decomposition by simultaneously measuring the velocity at 138 points on the $x/D = 3$ plane using hot-wire anemometers [1]. He found that the $4, 5$, and 6 modes seem to correspond to streamwise rib vorticies that occur in between the ring vorticies (*v* figure 1) similar to the streamwise vortices that occur in the planar mixing layers braid region (*cf* Rogers and Moser 1994). The experimental evidence also suggests that these modes may entrain more of the ambient air than the ring vorticies, as commonly assumed.

Although this experiment yielded very useful physical information about the dynamics of the flow, the effort required to perform the experiment effectively precludes its use on a wide variety of flows. The results of the experiment suggests, though, that it would useful to pursue alternative approaches that could yield reasonable estimates of the same information. One approach, suggested by Bonnet *et al.* (1994), is to measure the velocity at a small number of the necessary points and use Linear Stochastic Estimation to approximate velocity at the rest of the points. This estimate of the instantaneous velocity field could then be projected on the orthogonal basis to yield estimates of the instantaneous coefficients. These, in turn, could be used to produce partial reconstructions of the velocity field that model the dynamics of the large-scale motions.

[1] The main difficulty in determining the coefficients is that it is necessary to simultaneously measure at a sufficient number of points to accurately estimate the coefficients using equation 2 with aliasing information from the modes that are not of interest.



(a)



(b)

Figure 1: Instantaneous realizations from the partial reconstructions of the large-structures in the axisymmetric shear layer. Velocity countours correspond to different values of the streamwise fluctuating velocity. Figure (a) shows evidence of a ring like structure in the shear layer. Figure (b) shows evidence of streamise vorticies entraining ambient air to the core and advecting core fluid to the outside of the layer.

Bonnet *et al.* (1994) found that this technic produced reasonable estimates when it was applied with the POD to measurements in one homogeneous direction of a shear layer. For example, the technic was capable of accurately reproducing the dynamics of the ring vorticies in the annular shear layer (*cf* Ukeiley *et al.*, 1994) using measurements at only two radial positions in the shear layer. Bonnet *et al.* speculated that this technic could be expanded to experiments with more spatial directions, however, no attempt was made to test this conjecture.

The objective of this investigation is to determine whether this technic can be used to reproduce the dynamics on a full $r-\theta$ plane in the annular shear layer. The approximate technic of Bonnet *et al.* (1994) will be applied to the existing data base reported by Citriniti (1996) at $x/D = 3$ in the axisymmetric shear layer. The predictions of the approximate technic will be compare to the results reported by Citriniti (1996) to determine how well the technic reproduces the dynamics of both the ring structures and the streamwise structures in the braid region. Particular attention will be paid to the braid region since this region appears to play a dominant role in the entrainment process and the previous results (*i.e.*Ukeiley *et al.*, 1994) suggest that the approximate technic is capable of reproducing the dynamics of the ring vorticies. This information is very useful because it can then be used to apply the combined POD/LSE technic to other related flows that are currently being investigated by the authors, such as the swirling axisymmetric jet (Ewing and Pollard, 1996) or the ventilated three-dimensional wall jet (Benaissa *et al.*, 1996).

References

Benaissa, A., Cordier, F., Ewing, D., and Pollard, A. (1996), Investigation of a three-dimensional jet with axis removed from the wall, 49th Annual Meeting of the Division of Fluid Dynamics of the American Physical Society, Syracuse, NY, Nov. 24-26.

Bonnet, J. P., Cole, D. R., Deville, J., Glauser, M. N., and Ukeiley, L. S. (1994), Stochastic Estimation and Proper Orthogonal Decomposition: Complementary Techniques for Identifying Structure, *Expts. in Fluids*, **17**, 307-314.

Bonnet, J. P. and Glauser, M. N. (eds.) (1994) *Eddy Structure Identification in Free Turbulent Shear Flows*, Kluwer Academic Press.

Citriniti, J. C. (1996), Experimental Investigation into the of the Axisymmetric Mixing Layer Utilizing the Proper Orthogonal Decomposition., *Ph.D. Dissertation*, State University of New York at Buffalo, Amherst, NY.

Corke, T. C., Glauser, M. N., and Berkooz, G. (1994), Utilizing low-dimensional dynamical systems models to guide control experiments, *Proc. of the Twelfth US National Congress of Applied Mech.*, Seattle.

Ewing, D. and Pollard, A. (1996), 49th Annual Meeting of the Division of Fluid Dynamics of the American Physical Society, Syracuse, NY, Nov. 24-26.

Glauser, M. N., Lieb, S. J., and George, W. K. (1987), Coherent Structures in the Axisymmetric Jet Mixing Layer, *Turbulent Shear Flows 5*, eds. F. Durst *et al.*, Spinger-Verlag, 134.

Lumley, J. L. (1967), The structure of inhomogeneous turbulent flows. *In Atmospheric Turbulence and Radio Wave Propogation*, Nauka, Moscow.

Rogers, M. M. and Moser, R. D. (1994), Direct Simulation of a Self-Similar Turbulent Mixing Layer, *Phys. Fluids*, **6**(2), 903-923.

Ukeiley, L. S., Cole, D. R., and Glauser, M. N. (1994), An Examination of the Axisymmetric Jet Mixing Layer Using Coherent Detection Techniques, *In Eddy Stucture Identification in Free Turbulent Shear Flows*, eds. J. P. Bonnet and M. N. Glauser, Kluwer Academic Press, 325-336.