Polynomial Calibrations for Hot Wires in Thermally Varying Flows

William K. George
Paul D. Beuther
Aamir Shabbir
Turbulence Research Laboratory,
Department of Mechanical and Aerospace Engineering,
University at Buffalo, SUNY,
Buffalo, New York

■An alternative to the traditional forms for hot-wire calibrations is presented that expresses flow velocity as a function of voltage in the form of a polynomial $U = \Sigma_{n=0}^{N} A_n E^n$ or $\mathrm{Re} = \Sigma_{n=0}^{N} A_n' \, Nu^n$, where the coefficients are velocity-independent. These forms have the advantage that the velocity can be calculated directly (and recursively) from the measured voltages once the coefficients are determined—a significant advantage for computer implementation. Moreover, the coefficients occur linearly and can be determined by linear regression. Since the primary errors in calibration are usually in the determination of velocity whereas the voltage can be more accurately determined, the regression is properly directed to minimize the effect of measurement error, unlike the King's law expressions and polynomials expressing voltage as a function of velocity. Finally the quasi-linearization of the above expressions is discussed.

Keywords: *hot-wire anemometer, thermally varying flows*

INTRODUCTION

The purpose of this article is to discuss the use of a polynomial for hot-wire velocity calibration and to present some of its advantages over the more commonly used King's law expressions. While some of these advantages are well known (for example, ease of implementation on a computer), others are not so well known. Moreover, there is a natural hesitancy among many in the flow-measuring community to deviate from King's law linearization because of its long-standing acceptance. While this may be reasonable when using analog linearizers that may already be at the disposal of the investigators, it is suggested here that there are definite advantages to be gained by using a polynomial scheme—both for ease of implementation and for accuracy. This is especially true when implementing a digital linearization or using a quasi-linearization technique.

KING'S LAW

In 1906 King [1] obtained the following analytical solution to the problem of heat transfer from potential flow around a cylinder:

$$\mathrm{Nu} = A + B\,\mathrm{Re}^{1/2} \tag{1}$$

where

$$\mathrm{Nu} = hd/k \qquad \text{(Nusselt number)} \tag{2}$$

$$\mathrm{Re} = Ud/\nu \qquad \text{(Reynolds number)} \tag{3}$$

$$h = \frac{q_w}{T_w - T_a} \qquad \text{(heat transfer coefficient)} \tag{4}$$

and where d is the wire diameter, k is the thermal conductivity of the fluid, ν is the kinematic viscosity of the fluid, q_w is the heat transfer rate per unit area from the wire, and T_w and T_a are the wire surface and ambient fluid temperatures, respectively.

While potential flow around a cylinder has little to do with the flow around a typical hot wire ($10^{-2} < \mathrm{Re} < 100$), the form of King's law has been retained in many of the subsequent attempts to establish empirical laws. These were reviewed in detail by McAdams [2] and Collis and Williams [3]. A commonly used expression is a composite of these due to Kramers [4] and Collis and Williams (cf Hinze [5], namely,

$$\mathrm{Nu}\left(\frac{T_w}{T_f}\right)^m = A\,\mathrm{Pr}^p + B\,\mathrm{Pr}^q\,\mathrm{Re}^n \tag{5}$$

where Pr is the fluid Prandtl number and $(T_w/T_f)^m$ is a loading factor that compensates for the variation in the thermal properties of the fluid in the thermal boundary layer of the wire. Typically $m = 0.17$, $p = 0.2$, $q = 0.33$, and $0.45 \le n \le 0.5$, the latter depending on Reynolds number. Additional terms or alternative expressions are needed at very low Reynolds numbers where the character of the equations is dominated by viscosity and/or free convection. An example of the former is the linear term often added at low velocities; the latter is illustrated by the Oseen-type logarithmic calibration used by Collis and Williams [3] and George et al [6].

In rarefied gases or with very fine wires (less than 1 μm diameter is air), the fact that the wire diameter and the mean free path λ are of the same order of magnitude is responsible for a breakdown in the continuum approximation. This can give rise to a dependence of the calibration on Knudsen number $K = d/\lambda$ (see for example, Collis and Williams [3]). Additional problems can occur at very low velocities where natural convection from the wire can dominate convective

Address correspondence to Dr. Aamir Shabbir, Department of Mechanical and Aerospace Engineering, University of Buffalo, SUNY, Amherst, New York 14260.

Experimental Thermal and Fluid Science 1989; 2:230–235
Published 1989 by Elsevier Science Publishing Co., Inc., 655 Avenue of the Americas, New York, NY 10010 0894-1777/89/$0.00

effects, and effectively limit the lower range of applicability of the calibrations. These have been discussed in some detail by Hollasch and Gebhart [7] and Warpinski et al [8].

THE ADVANTAGES OF A POLYNOMIAL

While the above expressions (and others similar to them) can be used in a variety of applications with success, they are always awkward and difficult to implement—both to obtain the calibration curve and to apply it. The principal reason for this is the manner in which the voltage depends on the velocity. This is easily seen by the constant ambient temperature version of Eq. (5), which reduces for a given wire to

$$E^2 = A + BU^n \tag{6}$$

where E is the bridge top voltage typically measured in constant-temperature anemometer (CTA) applications.

Performing the calibration is complicated by the fact that the determination of A is not as simple as measuring E with $U = 0$ because of the natural convection effects mentioned above. This uncertainty is carried through the analysis since B and n can be determined only after A is determined and subtracted. Moreover, it should be noted that the usual linear least-squares type of analysis is not applicable to this expression—first, because it is not a linear equation and, second, because the principal uncertainty in the calibration measurements is usually in U and not in E.

Implementation of even the simple form of King's law given in Eq. (6) is not straightforward either. Calculating the velocities from the measured voltage requires a time-consuming process on a computer, and analog conversion requires sophisticated circuitry.

The idea and practice of using a polynomial in both digital and analog linearization is not new. Cheesewright [9], for example, discussed digitally linearizing hot-wire signals on a large computer, while commercial polynomial linearizers have been available for a number of years.

In its simplest form, the velocity is expressed as the sum of powers of the voltages, that is,

$$U = \sum_{n=0}^{N} A_n E^n \tag{7}$$

The principal advantages are twofold: First, a linear least-squares analysis an be directly applied since the coefficients occur linearly and since the principal uncertainty in the observations is on the left, namely U. Second, application of the calibration to measured data is straightforward and involves only recursive multiplication of the measured voltages.

An alternative form that is equivalent to Eq. (7) but is more convenient for analog implementation uses the offset voltage $E - E_{\text{ref}}$ instead; thus,

$$U = \sum_{n=0}^{N} A_n (E - E_{\text{ref}})^n \tag{8}$$

A good choice for E_{ref} is either the "zero" velocity voltage or a midrange voltage. [Note that for reasons that will become clear in the next section it is sometimes preferable to work with Eq. (7) even if an offset is used prior to digitization. This is easily accomplished by adding the offset to the numbers stored in the computer.]

Because of the convenience of using a linear least-squares or other statistical algorithm for determining the coefficients (these algorithms can be carried out even on a hand calculator), it is no longer necessary to try to infer or measure the anemometer output at zero velocity. Thus the calibration can be performed entirely in the region where the wire will be used, and the natural convection regime avoided entirely. (In fact, this is the manner in which any calibration should be used, regardless of curve fit employed.)

It is, of course, no surprise that polynomials can also provide superior fits to the calibration data, since by increasing the order of the polynomial the number of adjustable coefficients is also increased. However, our experience suggests that there is little to be gained by going beyond the fourth order, that is,

$$U = A_0 + A_1 E + A_2 E^2 + A_3 E^3 + A_4 E^4 \tag{9}$$

Independent studies by Wlezien [10] have also shown this choice to be superior over a wide range of velocities to other calibration commonly used methods.

A POLYNOMIAL HEAT TRANSFER LAW

Equations (7)–(9) have the principal disadvantage that their coefficients are temperature-dependent. This can be contrasted with Eqs. (1) and (5), which are presumed valid over a wide range of temperatures. There is, of course, no reason why a "heat transfer law" that is based on a power law cannot be postulated. Therefore we propose that the Reynolds number be expressed in half-powers of the Nusselt number, that is,

$$\text{Re} = \sum_{n=0}^{N} C_n \, \text{Nu}^{n/2} \tag{10}$$

The coefficients C_n are now temperature-*independent*, and the temperature dependence enters entirely through the variations of ν and k in the Nusselt and Reynolds numbers and through the direct dependence of Nu on $T_w - T_a$.

We have had great success with the fourth-order polynomials in $\text{Nu}^{1/2}$; that is,

$$\text{Re} = C_0 + C_1 \, \text{Nu}^{1/2} + C_2 \, \text{Nu} + C_3 \, \text{Nu}^{3/2} + C_4 \, \text{Nu}^2 \tag{11}$$

The Reynolds number is evaluated at the gas temperature, while the Nusselt number is evaluated at the film temperature $T_f = (T_w + T_a)/2$; that is, $\nu = \nu_a$, while $k = k_f$. [Note that $\text{Nu}^{1/2} \propto E$ for fixed temperatures, and thus Eq. (11) reduces to Eq. (9) for this case.]

An example of a single-wire calibration (Dantec type 55P76 gold-plated 5 μm wire) is shown in Fig. 1. A second example is the x-wire (Dantec custom-made probe, gold-plated 2.5 μm wires) data shown in Fig. 2. Both of these calibrations were carried out for conditions corresponding to those present in buoyant plume experiments where the local flow temperature varied over 20°C and where flow velocities ranged from 0.12 to 1.5 m/s. (Note that for the x-wire an angle calibration needs to be done in addition to the Re–$\text{Nu}^{n/2}$ calibration.)

The curves shown in Figs. 1 and 2 were obtained by a linear regression fit to the calibration data. Generally, it was possible to achieve a maximum relative deviation (between predicted and measured velocities) of 0.5%, which was well within the accuracy of the velocity measurement in the calibrator. Note that it is important in most cases to minimize the relative error

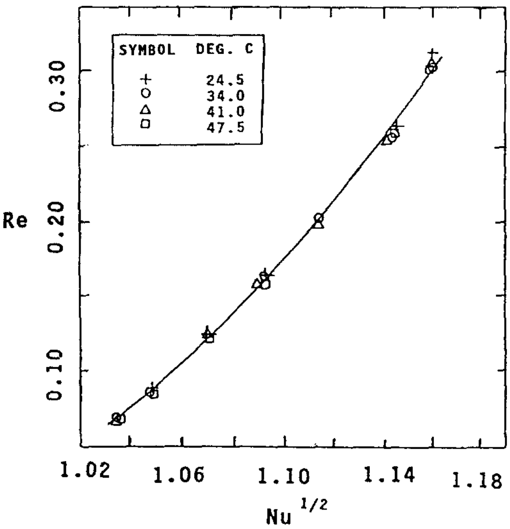

Figure 1. Calibration curve for the Dantec 55P76 probe.

in the curve fitting, and not the absolute error, since otherwise large errors can result at the lower velocities.

Because of slight differences in length and diameter, the calibrations for the two wires did not coincide. These parameters were not measured directly, and there was undoubtedly some deviation from the manufacturer's stated values.

Since the wires are calibrated individually, this is not important, although it would have been had a general result (as for a wire of infinite length) been desired. Two parameters that were difficult to measure directly were the sensor resistance and the coefficient of thermal resistivity (and hence the exact wire temperature)—the former because there was no sure way to short the probe without breaking it, and the latter because of a slight dependence on annealing history. Therefore these parameters were adjusted for each wire to give the best collapse of the data at all temperatures. A typical variation was less than 5%.

While there may be other polynomial expressions that can be used successfully, the use of polynomials in $\mathrm{Nu}^{1/2}$ is particularly convenient for digital analysis since $\mathrm{Nu}^{1/2} \approx E$, the anemometer bridge voltage. The kinematic viscosity and the thermal conductivity of air are conveniently calculated from the empirical expressions

$$\frac{\nu}{\nu_{\mathrm{ref}}} = \left(\frac{T}{T_{\mathrm{ref}}}\right)^{1.7} \tag{12}$$

and

$$\frac{k}{k_{\mathrm{ref}}} = \left(\frac{T}{T_{\mathrm{ref}}}\right)^{0.7} \tag{13}$$

or alternate expressions. The wire temperature T_w is fixed by the feedback amplifier (constant-temperature mode), and T_a can be separately measured by a parallel resistance wire, thermocouple, or thermistor. Note that care must be taken in the probe design to ensure that the thermal sensor is not contaminated by the thermal wake of the velocity wires, and yet all the sensors are close enough to be measuring the same fluctuation.

In the experiments of Beuther [11] and Shabbir [12] using x-wires, both the ambient temperature and the velocity anemometer bridge voltages were sampled simultaneously and Eq. (11) was used to calculate the instantaneous cooling velocities on each wire. This information was subsequently combined with the angle calibration to yield the two desired instantaneous velocity components. By first calculating $\mathrm{Nu}^{1/2}$ and then using recursive multiplication, the calculation could be rapidly performed.

The effect of including a temperature loading factor in the Nusselt number, i.e.,

$$\mathrm{Re} = \sum_{n=0}^{4} C_n \left[\mathrm{Nu}\left(\frac{T_w}{T_f}\right)^{0.17}\right]^{n/2} \tag{14}$$

has also been investigated. This approach was not as successful as the simpler and more direct approach outlined above and is not recommended.

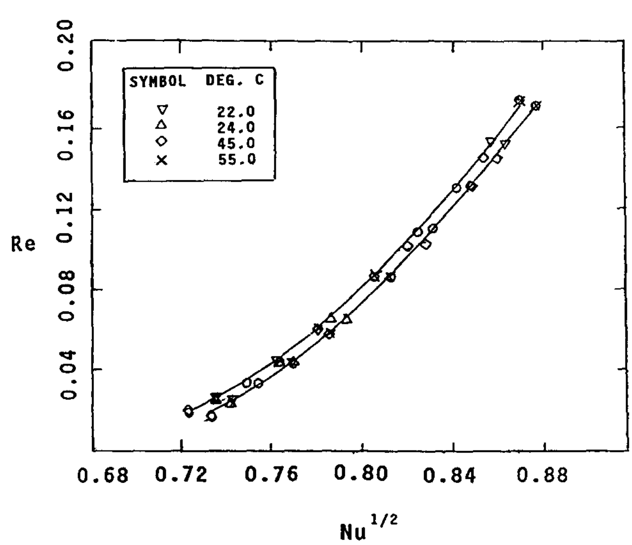

Figure 2. Calibration curve for the Dantec custom-made probe, which had an x-wire and a temperature wire.

APPLICATION TO THE QUASI-LINEAR TECHNIQUE

Sometimes analog linearization is avoided entirely and one works only with averaged signals. Such a procedure is called quasi-linearization. Due to the approximate nature of this method, it should not be used when one has access to a digital computer and an A/D converter. However, there are situations where the quasi-linearization technique can be useful, especially where on-line computer data acquisition is not possible.

The usual quasi-linearization begins with King's law type expressions. For example, by expanding Eq. (6) about average values, the following averaged equation is obtained:

$$\overline{(E+e)^2} = \bar{E}^2\left(1 + \frac{\overline{e^2}}{\bar{E}^2}\right) = A + \overline{B(U+u)^n}$$
$$\approx A + B\bar{U}^n\left(1 + \frac{n(n-1)}{2!}\frac{\overline{u^2}}{\bar{U}^2}\right) \tag{15}$$

where terms above $\overline{u^2}/\bar{U}^2$ have been neglected. By subtracting this equation from the instantaneous equation, squaring and averaging, expressions for $\bar{U}$ and $\overline{u^2}$ in terms of $\bar{E}$ and $\overline{e^2}$ can be obtained (cf Rao and Brzustowski [13]). Such expressions are, of course, valid only as long as $\overline{u^2}/\bar{U}^2$ is truly negligible; in other words, only in low-intensity turbulent flows.

Unfortunately, for many flows of interest, $\overline{u^2}/\bar{U}^2$ is not negligible—for example, the turbulent jet—and the only recourse is to linearize before averaging. The polynomial calibration scheme proposed above in Eqs. (7) and (10) open new possibilities for quasi-linearization that do not require that powers of the turbulence intensity remain small but impose only the less restrictive condition that powers of $\overline{e^2}/\bar{E}^2$ remain small. This represents a substantial improvement because typically

$$\frac{\overline{e^2}}{\bar{E}^2} \ll \frac{\overline{u^2}}{\bar{U}^2} \tag{16}$$

This is because of the fact that the anemometer set voltage $\bar{E}_0$, which is included in $\bar{E}$, is usually much larger than $\sqrt{\overline{e^2}}$.

From Eq. (9) for a constant-temperature flow, it follows from decomposing E and U into mean and fluctuating parts and averaging that

$$\bar{U} = A_0 + A_1\bar{E} + A_2(\bar{E}^2 + \overline{e^2}) + A_3(\bar{E}^3 + 3\bar{E}\overline{e^2} + \overline{e^3}) + A_4(\bar{E}^4 + 6\bar{E}^2\overline{e^2} + 4\bar{E}\overline{e^3} + \overline{e^4}) \tag{17}$$

By subtracting this from Eq. (9), an equation for the fluctuating velocity can be obtained as

$$u = A_1 e + A_2[2\bar{E}e + (e^2 - \overline{e^2})] + A_3[3\bar{E}^2 e + 3\bar{E}(e^2 - \overline{e^2}) + (e^3 - \overline{e^3})] + A_4[6\bar{E}^2(e^2 - \overline{e^2}) + 4\bar{E}^3 e + 4\bar{E}(e^3 - \overline{e^3}) + (e^4 - \overline{e^4})] \tag{18}$$

If all terms above the second order are assumed negligible, Eq. (17) for the mean velocity reduces to

$$\bar{U} \cong A_0 + A_1\bar{E} + A_2\bar{E}^2\left(1 + \frac{\overline{e^2}}{\bar{E}^2}\right) + A_3\bar{E}^3\left(1 + \frac{3\overline{e^2}}{\bar{E}^2}\right) + A_4\bar{E}^4\left(1 + \frac{6\overline{e^2}}{\bar{E}^2}\right) \tag{19}$$

Typically in the experiments described later, $\bar{E} \simeq 3.4$ V while $e_{\rm rms} \simeq 100$ mV, so the neglected terms are at most 3% of the second-order terms which are themselves less than 1% of the zeroth-order terms and could also be neglected.

By squaring and averaging Eq. (18) for the fluctuating velocity and again ignoring terms of order higher than the second, the mean square fluctuating velocity can be expressed as

$$\overline{u^2} \simeq [A_1 + A_2(2\bar{E}) + A_3(3\bar{E}^2) + A_4(4\bar{E}^3)]\overline{e^2} \tag{20}$$

Additional terms in $\overline{e^3}/\bar{E}^3$ could be retained; however, in many applications this seems unnecessary as this quantity is almost always small compared to $\overline{e^2}/\bar{E}^2$.

Equations (19) and (20) make it clear that accurate measurements of the mean and rms turbulent velocities can be made with the quasi-linearization technique by measuring only the mean and rms anemometer voltages. This is true even in flows of relatively high turbulence intensities ($u'/\bar{U} \approx 1$), because of the relatively low value of $e'/\bar{E}$. Thus, the polynomial linearization scheme possesses significant advantages when quasi-linearization techniques must be used.

QUASI-LINEARIZATION IN THE PRESENCE OF TEMPERATURE FLUCTUATIONS

The ideas expressed above can also be applied to the general heat transfer law, Eq. (11), to yield a quasi-linear result that is valid even when the temperature is also fluctuating. This follows by averaging Eq. (11) to obtain

$$\overline{\mathrm{Re}} = B_0 + B_1\,\overline{\mathrm{Nu}^{1/2}} + B_2\,\overline{\mathrm{Nu}} + B_3\overline{\mathrm{Nu}^{3/2}} + B_4\,\overline{\mathrm{Nu}^2} \tag{21}$$

If the temperature dependence of the fluid properties is ignored and the analysis is restricted to modest temperature fluctuations (relative to the absolute temperature, $\simeq$ 300 K typically), the Nusselt number can be written as

$$\mathrm{Nu} = \frac{C_1 E_u^2}{T_w - T} \tag{22}$$

where C_1 includes the missing terms from Eqs. (2) and (4) and is assumed constant, and E_u denote the anemometer output.

Expanding, we have

$$\mathrm{Nu} = C_1\frac{(\bar{E}_u + e_u)^2}{(T_w - \bar{T})[1 - t/(T_w - \bar{T})]} \tag{23}$$

where t represents the fluctuating temperature. Note that the average wire overheat ratio is given by $(T_w - \bar{T})/\bar{T}$ and is normally greater than 0.5.

By expanding the denominator using the binomial theorem and neglecting all terms of order higher than the second, it can easily be shown that the terms of Eq. (21) can be represented by the expression

$$\overline{\mathrm{Nu}^{n/2}} \cong \left(\frac{C_1\bar{E}_u^2}{T_w - \bar{T}}\right)^{n/2} \cdot \left[1 + \frac{n(n-1)}{2}\left(\frac{\overline{e_u^2}}{\bar{E}_u^2}\right) + \frac{n^2}{2}\left(\frac{\overline{e_u e_t}}{(T_w - \bar{T})\bar{E}_u}\right)\right] \tag{24}$$

from which the Reynolds number can be readily evaluated. The mean velocity can be calculated from

$$\bar{U} = \frac{\nu}{d}\{\overline{\mathrm{Re}}\} \tag{25}$$

By subtracting (21) from (11), an equation for the fluctuating Reynolds number ud/ν can be obtained. After squaring and averaging, an equation for $\overline{u^2}$ can be obtained. The final result is

$$\begin{aligned}\sqrt{\overline{u^2}} = \frac{\nu}{d}\{&B_1^2[\overline{\mathrm{Nu}} - (\overline{\mathrm{Nu}^{1/2}})^2] + B_2^2[\overline{\mathrm{Nu}^2} - (\overline{\mathrm{Nu}^2})^2] \\ &+ B_3^2[\overline{\mathrm{Nu}^3} - (\overline{\mathrm{Nu}^{3/2}})^2] + B_4^2[\overline{\mathrm{Nu}^4} - (\overline{\mathrm{Nu}^2})^2] \\ &+ 2B_1B_2(\overline{\mathrm{Nu}^{3/2}} - \overline{\mathrm{Nu}^{1/2}}\,\overline{\mathrm{Nu}}) + 2B_1B_3(\overline{\mathrm{Nu}^2} - \overline{\mathrm{Nu}^{1/2}}\,\overline{\mathrm{Nu}^{3/2}}) \\ &+ 2B_1B_4(\overline{\mathrm{Nu}^{5/2}} - \overline{\mathrm{Nu}^{1/2}}\,\overline{\mathrm{Nu}^2}) + 2B_2B_3(\overline{\mathrm{Nu}^{5/2}} - \overline{\mathrm{Nu}}\,\overline{\mathrm{Nu}^{3/2}}) \\ &+ 2B_2B_4(\overline{\mathrm{Nu}^3} - \overline{\mathrm{Nu}}\,\overline{\mathrm{Nu}^2}) + 2B_3B_4(\overline{\mathrm{Nu}^{7/2}} - \overline{\mathrm{Nu}^{3/2}}\,\overline{\mathrm{Nu}^2})\}\end{aligned} \tag{26}$$

To obtain an expression for the heat flux $\overline{ut}$, first equations for the fluctuating temperature and fluctuating velocity are obtained. These are then multiplied and averaged to obtain

$$\overline{ut} = \frac{\nu A_2}{d}(B_1\,\overline{\mathrm{Nu}^{1/2}\,e_t} + B_2\,\overline{\mathrm{Nu}\,e_t} + B_3\,\overline{\mathrm{Nu}^{3/2}\,e_t} + B_4\,\overline{\mathrm{Nu}^2\,e_t}) \tag{27}$$

Table 1. Comparison of Quasi-linearization Results with Those Computed from Instantaneous Measurements in a Turbulent Plane

	$U(m/s)$			$\overline{ut}$		
u'/U	*Digital Technique*	*Quasi-linearization*	*Percent Error*	*Digital Technique*	*Quasi-linearization*	*Percent Error*
0.330	0.7468	0.7185	−6.05	0.881	0.824	−6.46
0.336	0.7650	0.7259	−5.10	1.145	1.091	−4.41
0.476	0.558	0.518	−7.16	1.398	1.282	−8.29
0.652	0.342	0.328	−4.10	0.883	0.776	−12.1
0.940	0.115	0.1086	−5.50	0.174	0.128	−26.0

Where $\overline{\mathrm{Nu}^{n/2}e_t}$ can be obtained by multiplying the expression for $\mathrm{Nu}^{n/2}$ by e_t and then averaging. The result is

$$\overline{\mathrm{Nu}^{n/2}\,e_t}=n\left(\frac{C_1}{T_w-\bar{T}}\right)^{n/2}\left(\overline{e_u e_t}\bar{E}_u^{n-1}+\frac{\bar{E}_u^n\overline{e_t^2}}{2(T_w-T)}\right) \quad (28)$$

The constant A_2 in Eq. (27) is the calibration constant for the temperature wire, ie,

$$T=A_1+A_2e_t$$

Thus a single anemometer used in conjunction with a fast thermometer (eg, a resistance wire), d.c. and rms voltmeters, and either a multiplication circuit or summing or differencing amplifiers can yield accurate measurement of not only $\bar{U}$ and $\overline{u^2}$ but also $\bar{T}$, $\overline{t^2}$, and $\overline{ut}$, even when the turbulence intensity is relatively high. A procedure for achieving this was implemented by Ahmed [14].

Table I shows an evaluation of the quasi-linearization method based on Eq. (11). The results were obtained in a buoyant plume by direct linearization of the data using Eq. (11), and by quasi-linearization. The digital technique is considered standard for comparison purposes. Even the second-order moments are seen to be accurately measured for all but the highest turbulence intensities, a real surprise considering the total disregard of the third moments.

USEFULNESS

A method for hot-wire calibrations has been presented that expresses flow velocity as a polynomial function of voltage. For a nonisothermal flow the Reynolds number is expressed as a function of the Nusselt number. Since primary errors during calibration are in the determination of velocity and not of voltage, the linear regression for such polynomials is properly directed to minimize the effect of measurement error. Another advantage of using such calibration relations is that there is no longer a need to infer the anemometer output for zero velocity.

Quasi-linearization of the above relations allows the calculation of the mean and turbulent quantities from the averaged anemometer output. For turbulence intensities of up to 50%, the error in the calculation of second moments was only 10%. However, it should be kept in mind that due to the approximate nature of this method, it should be used only when an on-line computer and an A/D converter are not available.

The work reported here formed a portion of the Ph.D. dissertations of P. D. Beuther and A. Shabbir. The research was supported by the National Science Foundation, Division of Atmospheric Sciences under grants ATM 78 08442 and ATM 80 23699A01.

NOMENCLATURE

A	empirical coefficient, dimensionless
A_n	empirical coefficient in Eqs. (7), (8), dimensionless
B	empirical coefficient, dimensionless
C_n	empirical coefficients in Eq. (10), dimensionless
d	wire diameter, m
E	CTA output voltage, V
E_0	CTA output voltage at hypothetical zero velocity, V
h	heat transfer coefficient defined by Eq. (4), W/(m K)
k	thermal conductivity of fluid, W/(m K)
K	Knudsen number ($=d/\lambda$), dimensionless
m	exponent in temperature loading factor, Eqs. (5), (14), dimensionless
Nu	Nusselt number defined by Eq. (2), dimensionless
n	empirical exponent, dimensionless
P	empirically determined exponent in Eq. (5), dimensionless
q	empirically determined exponent in Eq. (5), dimensionless
q_w	heat transfer rate per unit area from wire, W/m^2
Re	Reynolds number defined by Eq. (2), dimensionless
t	fluctuating ambient temperature, °C
T_f	film temperature [$=(T_w+T_a)/2$], °C
T_w	wire temperature, °C
T, T_a	temperature of ambient fluid, °C
ν	kinematic viscosity, m/s
λ	mean free path, m

REFERENCES

1. King, L. S., On the Convection Heat Transfer from Small Cylinders in a Stream of Fluids. Determination of Convection Constants of Small Platinum Wires with Application to Hot-Wire Anemometry, *Phil. Trans. Roy. Soc. London,* **214A**, 373–432, 1914.
2. McAdams, N.H., *Heat Transmission,* McGraw-Hill, New York, 1954.
3. Collis, D. L., and Williams, M. J., Two Dimensional Convection from Heated Wires at Low Reynolds Number, *J. Fluid Mech.*, **6**, 357–384, 1959.
4. Kramers, H., Heat Transfer from Spheres to Flowing Media, *Physica*, **12**, 61–120, 1946.
5. Hinze, J. O., *Turbulence*, McGraw-Hill, New York, 1975.
6. George, W.K., Alpert, R. L., and Tamanini, F., Turbulence Measurements in an Axisymmetric Buoyant Plume, *Int. J. Heat Mass Transfer,* **20**, 1145–1153, 1977.
7. Hollasch, K., and Gebhart, B., Calibration of Constant Temperature Hot-Wire Anemometers at Low Velocities in Water with Variable Fluid Temperature, *J. Heat Transfer*, **94**(1), 17–22, 1972.
8. Warpinski, N. R., Nagib, H. M. and Lavan Z., Experimental

Investigation of Recirculating Cells in Laminar Coaxial Jets, *AIAA J.*, **10**(9), 1204–1210, 1972.
9. Cheesewright, R., *Proc. of the DISA Conference on Flow in Industrial and Medical Environment,* David Cockrell, Eds., Univ. of Leicester Press, Leicester, England, April 1972.
10. Wlezien, R. W., *Bull. Am. Phys. Soc.,* **24**, 1142, 1979.
11. Beuther, P. B., Experimental Investigation of the Axisymmetric Turbulent Buoyant Plume, Ph.D. dissertation, SUNY at Buffalo, 1980.
12. Shabbir, A., An Experimental Study of an Axisymmetric Turbulent Buoyant Plume and Evaluation of Closure Hypotheses, Ph.D. dissertation, SUNY at Buffalo, 1987.
13. Rao, V. K., and Brzustowski, T. A., Preliminary Hot-Wire Measurements in Free Convection Zones over Model Fires, *Combust. Sci. Technol.,* **1**, 171–180, 1969.
14. Ahmad, M., An Experimental Study of an Axisymmetric Turbulent Buoyant Plume, M.S. thesis, SUNY at Buffalo, 1980.

Received April 18, 1988; revised December 6, 1988