An Application of the Proper Orthogonal Decomposition to the Axisymmetric Mixing Layer. Part 1: Determination of the POD Eigenfunctions.

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A proper orthogonal decomposition is used to extract eigenvectors from two-point velocity measurements in the mixing layer of a high Reynolds number axisymmetric jet. Cross-spectra were measured over an $8x8x48x512$ grid at one streamwise location, $x/D = 3$. Application of the proper orthogonal decomposition in the radial direction yields the lowest order eigenfunction which contains 40 percent of the turbulent kinetic energy and typically 85 percent of the various Reynolds stresses in the jet mixing layer. It was possible to almost completely reconstruct instantaneous signals of the streamwise and radial velocity components in the jet mixing layer from these proper orthogonal eigenfunctions using only the first few modes.

Further decomposition in the azimuthal direction, utilizing the harmonic decomposition, reveals the existence of a coherent ring-like structure dominated by the axisymmetric mode near the potential core, but with the fourth, fifth and sixth azimuthal modes playing a significant role from the center of the mixing layer and outward toward the low-speed side of the mixing layer. The Reynolds stress azimuthal correlations and their breakdown into azimuthal modes show that the incoherent turbulence is concentrated in the center of the coherent structure, indicating that it has either been generated or advected there.

From the results of the proper orthogonal decomposition and the azimuthal harmonic decomposition a life cycle is proposed for the evolution of coherent structures in the jet mixing layer. This life cycle begins with inception of rings of concentrated vorticity from an instability of the mean flow. A mutual interaction then occurs between two different rings, analogous to the first stages of the leapfrogging type phenomenon. A vortex instability arises from the further interaction of these two rings which finally results in a cascade of energy to smaller scales by vortex breakup and stretching.

1. INTRODUCTION

1.1. *Historical Review*

The large scale features of turbulent flows are obvious to even the most casual observer of nature. In spite of this, however, the effort to quantify these coherent and clearly visible aspects of turbulent motions has proven to be a considerable challenge to the scientific community. In no flow has the evidence for the existence of coherent structures been more tantalizing than the axisymmetric jet mixing layer, nor is there any flow where they are potentially more important to an understanding of its dynamics. Yet even in this flow, the coherent structures can not be unambiguously identified, much less their dynamics understood.

The first efforts to systematically study the large scale eddies of turbulent shear flows were carried out by Townsend and his students who postulated them to explain the slow roll-off of measured velocity correlations at large time and space separations (v. Townsend 1956, Grant 1958). The large eddies were assumed to be responsible for the finite correlation at lags bigger than the integral scale and to account for 10-20 percent of the total energy. They were also believed to be responsible for intermittency and entrainment, both prominent features of most external turbulent shear flows. However, they were not believed to be very involved (at least directly) in the energetics of the flow, and thus were viewed as rather passive contributors to the dynamics of the turbulence.

In the early 1970's the concept of *coherent structures* was introduced (v. Kovasznay et al 1970, Crow and Champagne 1971, Brown and Roshko 1974). These coherent structures were believed to be very active phenomena. They were clearly visible with flow visualization (at least at low Reynolds numbers) where they were seen to pair and explode (v. Hussain and Clark 1981). Like Townsend's (1956) big eddies these coherent structures were also believed responsible for entrainment and mixing. The response of the turbulence community to these observations was immediate and enthusiastic. It was suggested that turbulence could not be modeled, much less understood, without explicitly accounting for the existence of coherent structures (Liepmann 1979). Averaging, at least of the conventional form, was considered outdated since it smeared out such events and in this view only conditional averaging could account for the existence of such events (Cantwell 1981). The conditional averaging approach has been used extensively over the past two decades (v. Hussain 1986 for a comprehensive review).

Where has this conditional sampling approach taken us? We have seen many coherent structures but we still do not have a very good idea as to why they arise nor do we understand their dynamics. In part this failure can be traced to the conditional sampling approach which while finding structures which resemble those we see, gives no information as to their dynamical role nor insight into how to write equations for them. Adrian, Moin and Moser (1987) have shown that these classical conditional averages yield little that is not available from the two point correlation tensor. They argue that if a structure, no matter how intermittent, contributes a majority of the total integrated energy or Reynolds stress, then it will dominate the two-point correlation statistics and therefore information about the structure will be retained in the correlation tensor. In fact, they show that linear stochastic estimates of quadrant II-type events calculated from the two-point correlation tensor agree quite well with the actual conditional averages.

In an attempt to provide an objective basis for finding the large eddies from large separation correlation measurements, Lumley (1967) proposed that a large eddy be identified by correlating its velocity with an ensemble of realizations of the flow field. Choosing the large eddy so that the mean square energy is maximized leads to a well-defined integral eigenvalue problem which has as its kernel the velocity cross-correlation tensor. Because this velocity tensor is symmetric the solutions to the integral eigenvalue problem are governed by the Hilbert-Schmidt theory (Lumley 1967). Lumley further proposed that the large eddy be identified as the least order eigenfunction from the proper orthogonal decomposition. It is interesting to note that by deciding at the beginning that the coherent structure should optimally represent the field in the mean square sense, the functions have been determined. As pointed out by George (1988) and Moser (1988), it actually matters not whether the field represents velocity (as in this case), vorticity, pressure, or temperature, the integral eigenvalue problem will simply have as its kernel the appropriate correlation tensor. Thus all of the subjectivity has been lumped into this projection and the choice of principle behind it. (George 1988 also notes that it might be interesting to explore the consequences of alternative choices for maximization and Moser 1988 develops these ideas using the vorticity field.)

Because the eigenfunctions arising from the proper orthogonal decomposition are orthogonal and form a complete set, Lumley noted that these solutions allow a representation of any one of the original realizations in terms of the orthogonal eigenfunctions extracted from the eigenvalue problem. It also allows for objective determination of the contributions of the individual orthogonal eigenmodes to the kinetic energies and Reynolds stresses. Thus, if one agrees to work with Lumley's definition then one can write equations to study the dynamics of these structures as suggested by Lumley (1967). For a review of recent work which utilizes the POD in this manner see Berkooz et al (1993).

Until very recently, only a small number of researchers have applied the proper orthogonal decomposition to various flows. Payne (1966) examined the wake of a circular cylinder utilizing the correlation measurements of Grant (1958). Because only the trace of the tensor was measured by Grant, a mixing length assumption was used in conjunction with the continuity equation to obtain the remaining terms in the correlation tensor. Bakewell and Lumley (1967) examined the near wall region of a turbulent pipe flow. In a recent review, George (1988) discusses the severe limitations of the early efforts, and points out that it is only with the recent experiments of Herzog (1986) in the turbulent pipe flow, earlier versions of the work reported here (Leib et al 1984, Glauser et al 1985,1987 and Glauser and George 1987a,b), and the numerical studies of Moin (1984), that the potential of the proper orthogonal decomposition is beginning to be realized.

Since the work described above, there has been a flurry of activity using proper orthogonal decomposition techniques, in part driven by interest in chaos and its relation to turbulence. Aubry et al (1988) utilized the eigenfunctions of Herzog (1986) as a *good* set of basis functions in a dynamical systems approach to the near wall region. Their results are not inconsistent with events such as bursting seen in experimental work. Deane et al (1991) developed POD based models for grooved channels and circular cylinders. They investigated the ability of these models to mimic full simulations for Reynolds numbers greater than from which the eigenfunctions were obtained. For the grooved channel they found that the models extrapolated quite well for several Reynolds numbers. For the cylinder wake, however, the models were found to be only valid in a Reynolds number region close to where the eigenfunctions were obtained. Glauser et al (1991, 1992) and Zheng (1991) have used the eigenfunctions described in this paper to develop a similar type of dynamical systems model for the jet mixing layer. They find clear evidence of pairs of vortices interacting in the streamwise direction resulting in a transfer of azimuthal to streamwise vorticity. Rajaee et al (1994) use the snapshot form of the POD to develop a low-dimensional model for a weakly perturbed free shear layer. They find that their simulations compare well with the direct projections of the snapshots on the eigenfunctions. Chambers et al (1988) and Sirovich and Rodriquez (1987) have successively applied this approach to Burgers equation and the Ginsburg-Landau equation respectively. Although these two applications are not in turbulent flows, both of the equations exhibit chaotic dynamics. A recent Cambridge monograph by Holmes, Lumley and Berkooz (Holmes et al. 1996) provides a nice overview on how such systems can be developed.

Moin (1984) and Moin and Moser (1989) utilized channel flow simulations to provide the two-point correlation tensor. They found the dominant eddy contributes as much as 76 percent to the turbulent kinetic energy. Sirovich et al (1987) used this approach to study turbulent Bernard convection. They found that the first 5 eigenvalues account for over 60 percent of the energy. Glezer et al (1989) applied an extended version of the POD to a time periodically forced mixing layer. Delville et al (1991) applied the proper orthogonal decomposition to a plane fully turbulent mixing layer. They found that 70 percent of the mean square streamwise velocity was contained in the first 3 modes. Ukeiley et al (1992) examined the multifractal character of the POD reconstructions of the instantaneous streamwise velocity fields in a lobed mixer flow. They found that the higher POD mode contributions to the fluctuating velocity field were more multifractal in character, indicative of smaller scales. Application of the POD to the axisymmetric mixing layer to study the large scale structure dynamics was performed by Citriniti and George (1997) (herein referred to as II). They showed structure dynamics in the layer and suggested a life cycle for the structure evolution. For a more comprehensive review of POD applications see Berkooz et al (1993).

The earliest investigations of the statistical characteristics of the jet shear layer were carried out by Laurence (1956), Davies et al (1963), Bradshaw et al (1964) and Crow and Champagne (1971). Subsequent to these studies were numerous attempts to study the coherent structures by conditional sampling techniques. Cantwell (1981), Hussain (1983,1986), Ho and Huerre (1984) and Liu (1989) provide extensive reviews of these and other efforts which confirmed the probable existence of coherent structures and gave some hint as to their character. The azimuthal velocity correlations and their subsequent breakdown versus azimuthal Fourier modes were studied by Sreenivasan (1984) and Hussain and Zaman (1980). Sreenivasan examined temperature and streamwise velocity azimuthal correlations for several radial and streamwise locations, and suggested that azimuthal mode number 6 may be important. Hussain and Zaman measured the azimuthal correlations of the fluctuating streamwise velocity for excited and unexcited jets at 3 streamwise locations in the near field shear layer and 3 radial locations. They noticed a stronger azimuthal correlation for the excited jet. Glauser and George (1987b) presented the first measurements of Reynolds stress azimuthal correlations and their subsequent breakdown versus azimuthal Fourier modes. Their results are qualitatively consistent with the afore-mentioned results and will be discussed in more detail in the *Results* section of this paper. Corke et al (1991) utilize higher order spectra to study mode selection and resonant phase locking in unstable axisymmetric jets. They show the importance of various helical modes in the jet mixing layer, consistent with what is observed in the work reported here. Long and Arndt (1985), Long et al (1993) and Long, Arndt and Glauser (1997) report the application of the POD to the pressure field in the jet mixing layer. They perform the POD in the streamwise direction for several azimuthal modes. They find that the eigenfunctions exhibit streamwise structure similar to wave packets.

All of these experiments (except those of Glauser and George 1987b), contained much less information about the two-point velocity correlation tensor than is needed to solve the integral eigenvalue problem proposed by Lumley. Hence the goal of the our work has been to acquire the necessary data to apply the proper orthogonal decomposition to the near field jet mixing layer, and to carry out and evaluate the decomposition as a means to identifying the role of coherent structures. The axisymmetric jet mixing layer study reported herein has been in progress for more than a decade, and is an outgrowth of an effort initiated in 1972 to understand the role of coherent structures in noise generation (v. Arndt and George 1974). The early experimental results (Khwaja 1981) established that the low wavenumber cross-spectra and correlations at large lags could be regarded as nearly self-preserving so that attention could be focused on a single streamwise location. Leib et al (1984) reported the first application of the proper orthogonal decomposition to the jet mixing layer, and confirmed the expected rapid convergence with only three eigenfunctions required to capture 95 percent of the streamwise velocity spectra. Subsequent studies reported by Glauser et al (1985,1987) demonstrated (for the first time) the ability of the proper orthogonal decomposition to reconstruct the details of the instantaneous signals. Additional measurements of the azimuthal variation of the eigenfunctions and multiple components of velocity were discussed by Glauser and George (1987a,b).

The information gained in these earlier studies was used to design the present experiment. The new results allow two separate applications of the POD to be performed: the first, an application of the proper orthogonal decomposition to a cross-section of the flow at a single streamwise location of the mixing layer; and the second, a complete three-dimensional space application of the POD at an instant in time. These applications will be described in detail below, following a review of the theory and a description of the experiment.

2. The Proper Orthogonal Decomposition

Following Lumley (1967), a deterministic structure, $\phi_i(\vec{x},t)$, is sought which has the largest mean square projection on the random velocity field, $u_i(\vec{x},t)$. Maximizing the mean square projection via the calculus of variations leads to the integral eigenvalue problem

$$\lambda\phi_i(\vec{x},t) = \int R_{ij}(\vec{x},\vec{x}',t,t')\phi_j(\vec{x}',t')d\vec{x}'dt'. \tag{2.1}$$

The symmetric kernel of this Fredholm integral equation is the two–point correlation tenser R_{ij} defined by

$$R_{ij}(\vec{x},\vec{x}',t,t') = \overline{u_i(\vec{x},t)u_j(\vec{x}',t')}, \tag{2.2}$$

where $\vec{x}$ and $\vec{x}'$ represent different spatial points and t and t' different times and the overbar denotes the appropriate average for the problem under consideration (see Section 3 for a discussion on how averaging was performed in the jet).

From the Hilbert–Schmidt theory it can be shown that the solution of the Fredholm integral equation shown in equation 2.1 for a symmetric kernel with finite total energy (i.e. statistically inhomogeneous and non-stationary), is a discrete set (v. Lumley 1970); hence equation 2.1 becomes

$$\lambda^n\phi_i^n(\vec{x},t) = \int R_{ij}(\vec{x},\vec{x}',t,t')\phi_j^n(\vec{x}',t')d\vec{x}'dt'. \tag{2.3}$$

The eigenfunctions of the Fredholm equation are orthogonal over the interval and

$$\int \phi_i^n(\vec{x},t)\phi_i^m(\vec{x},t)d\vec{x}dt = \delta_{nm} \tag{2.4}$$

for normalized eigenfunctions. The eigenvalues of the Fredholm equation with a real symmetric kernel are all real and uncorrelated, i.e.

$$\overline{a^n a^m} = \lambda^n \delta_{nm} \tag{2.5}$$

so that the fluctuating random field can be reconstructed from the eigenfunctions by

$$\vec{u}_i(\vec{x},t) = \sum_{n=0}^{N=\infty} a^n \phi_i^n(\vec{x},t). \tag{2.6}$$

The random coefficients can be calculated from

$$a^n = \int \vec{u}_i(\vec{x},t)\phi_i^{*n}(\vec{x},t)d\vec{x},dt \tag{2.7}$$

where the ϕ_i^n are the eigenfunctions obtained from equation 2.3. Using these, the two-point velocity correlation can be reconstructed as

$$R_{ij}(\vec{x},\vec{x}^{'},t,t^{'}) = \sum_{n=0}^{\infty} \lambda^n \phi_i^n(\vec{x},t)\phi_j^{*n}(\vec{x}^{'},t^{'}) \tag{2.8}$$

Thus, each eigenfunction makes an independent contribution to the kinetic energy, Reynolds stress and spectra. It follows from this and the orthogonality of the eigenfunctions that the total turbulent kinetic energy in the flow is given by the sum of the eigenvalues, λ^n.

In summary then, the basic idea behind the proper orthogonal decomposition is that one tries to optimally represent a random field by a set of deterministic functions which in turn are determined by the field itself. This is quite unlike the more common situation where one chooses a set of orthogonal functions (like the harmonic ones of Fourier analysis) and then seeks the coefficients necessary to represent the field. For the proper orthogonal decomposition, both the functions and the coefficients arise from the statistical properties of the random field itself. For more details on the proper orthogonal decomposition see Berkooz et al (1993), Lumley (1967, 1970), George (1988), and Moin and Moser (1989).

2.1. *Harmonic Decomposition*

If the random field is statistically homogeneous or periodic in one or more directions or stationary in time, the eigenfunctions become Fourier modes (Lumley 1967, 1970, George 1988), so that the proper orthogonal decomposition reduces to the harmonic decomposition in these directions. In the jet mixing layer to be studied here, the flow is periodic in the azimuthal direction and stationary in time. Since the eigenfunctions are known in these coordinates, it is convenient to first decompose the field into the appropriate Fourier modes, then apply the proper orthogonal decomposition to the Fourier coefficients. This procedure is described in detail in George (1988). First suggested by Lumley (1967), this is simply a consequence of the factorization of the eigenfunctions. The quantity of interest is then the cross-spectral tensor given by

$$B_{ij}(x,x';r,r';m,f) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{0}^{2\pi} R_{ij}(x,x';r,r';\vartheta,\tau)e^{-i(2\pi f\tau+m\vartheta)}d\tau d\vartheta \tag{2.9}$$

where f is the frequency (corresponding to the time lag $\tau = t' - t$) and m is azimuthal mode number (corresponding to the angular separation variable $\vartheta = \theta^{'} - \theta$). Note that f is a continuous variable as a consequence of the stationarity, while $m = \pm 1, \pm 2, \cdots$ since the flow is homogeneous and periodic in θ.

The eigenvalue problem corresponding to equation 2.3 can now be written in cylindrical coordinates for a fixed value of $x = x' = \overline{x}$ as

$$\lambda^n(m,f)\phi_i^n(\overline{x},r,m,f) = \int_R B_{ij}(\overline{x},\overline{x},r,r',m,f)\phi_j^n(\overline{x},r',m,f)r'dr'. \qquad (2.10)$$

where r and r' represent different spatial locations in the radial direction (i.e. the inhomogeneous direction). The domain over which the integration in equation 2.10 is to be performed is the shear layer which is presumed of finite total energy. Note that $\overline{x}$ is not a variable, but a fixed location, so that the decomposition is only a partial decomposition over three of the four variables. Note also that the eigenvalues and eigenfunctions are now a function of azimuthal mode number m and frequency f.

2.2. *The Decomposition in the Streamwise Direction*

In the results presented here the flow is assumed homogeneous in the streamwise direction so that it is possible to extract the streamwise variation and the unmeasured components of the correlation tensor upon application of the continuity equation as discussed in the next section. It should be noted, however, that this does *not* compromise the quality of the original measurements at $x/d = 3$ since they were obtained in the *growing* jet shear layer.

The wavenumber spectra are determined from the measurements of the frequency spectra using Taylor's hypothesis since the data was taken at a single streamwise location. However, when this is done (as will be the case below), it will not be possible to separately account for the temporal and streamwise spatial variation of the eigenfunctions. Thus, either the eigenvalue problem can be solved including time for a fixed value of x (as described in the previous section and in (II)), or the entire spatial variation can be obtained for a fixed value of time. Both these possibilities will be examined in the analysis of the data which follows.

The frequency dependence of the cross-spectra can be mapped into a wavenumber dependence resulting in

$$B_{ij}(r,r';f,m) \Rightarrow A_{ij}(r,r';k_1,m) \qquad (2.11)$$

where

$$k_1 = \frac{2\pi f}{U_c}. \qquad (2.12)$$

The convection velocity U_c was chosen to be constant across the layer at a value equal to $0.6U_e$. This is consistent with both the results of Zaman and Hussain (1981) who showed that a single convection velocity across the layer gave the best results for the large scale motions and with (II) where the structures were shown to be convected at this speed.

Thus by using Taylor's hypothesis in conjunction with the assumption of statistical homogeneity in x, the full spatial variation of the cross-spectral tensor can be obtained. This, together with the knowledge of the eigenfunctions in the azimuthal direction allows the two-point cross correlation to be reconstructed as

$$R_{ij}(\rho;r,r';\vartheta) = \sum_m \int_{-\infty}^{\infty} e^{i(k_1\rho+m\vartheta)} A_{ij}(r,r';k_1,m)dk_1, \qquad (2.13)$$

where A_{ij} is the kernel for inhomogeneous eigenvalue problem. Note now that A_{ij} is a function of k_1 and not f so that the eigenvalue problem solved becomes

$$\lambda^n(m,k_1)\phi_i^n(r,m,k_1) = \int_R A_{ij}(r,r',m,k_1)\phi_j^n(r',m,k_1)r'dr'. \qquad (2.14)$$

This decomposition can be contrasted with the time - dependent decomposition of equation 2.10. Both will be employed in analyzing the experimental data.

2.3. *Consequences of the Continuity Equation*

In the experiments reported below, only four of the nine components of the two-point cross-spectral tensor were determined. The remaining components are obtained from the appropriate symmetries and by application of the continuity equation. The continuity equation can be written in terms of R_{ij} as

$$\frac{\partial R_{ij}}{\partial x_i} = 0. \tag{2.15}$$

The correlation tensor can be represented as the Fourier inverse transform of A_{ij} as shown in equation 2.13 so that the continuity equation in cylindrical coordinates becomes

$$k_1 i A_{1j} + \frac{\partial A_{2j}}{\partial r} + A_{2j}/r - \frac{m}{ri} A_{3j} = 0. \tag{2.16}$$

This can be solved for A_{3j} to obtain

$$A_{3j} = \frac{ri}{m}[k_1 i A_{1j} + \frac{\partial A_{2j}}{\partial r} + A_{2j}/r], j = 1, 2, 3 \tag{2.17}$$

where A_{1j}, A_{2j} for $j = 1, 2$ are given from the experimental measurements. Equation 2.17 was solved numerically and the values of A_{ij} used in equation 2.14 to solve for the eigenvalues and eigenfunctions.

2.4. *Azimuthal symmetries in R_{ij} and A_{ij}*

The jet shear layer is symmetric in the azimuthal direction. Hence $R_{ij}(\vartheta) = R_{ij}(-\vartheta)$ when $i \neq 3$ and $j \neq 3$ or when $i = j = 3$. For the remaining combinations, $i, j =$ 1,3; 2,3; 3,2 and 3,1, $R_{ij}(\theta) = -R_{ij}(-\theta)$. The fact that these remaining combinations are odd can easily be shown from the continuity equation, written in terms of R_{ij}. Because R_{ij} has these symmetries and is real-valued, A_{ij}, is an even function of m for $i, j =$ 1,1; 1,2; 2,1; 2,2 and 3,3. For the other combinations, $i, j = 2, 3; 3, 2; 1, 3$ and $3, 1$, A_{ij} is an odd function of m. These symmetries were utilized to reduce by a factor - of - two the amount of data taken in the azimuthal direction, i.e., measurements where made from 0 to π only. Moin and Moser (1989) utilized similar symmetries for the spanwise direction in a channel flow simulation to reduce by a factor-of-two the computations required. Herzog (1986) utilized these symmetries in a pipe flow to find the structure of the solutions of the eigenvalue problem in wavenumber space.

3. Experiment

3.1. *An Overview of the Experimental Approach*

The axisymmetric jet mixing layer is a relatively simple flow to generate (based on previous experience) and the limitations of the stationary hot wires are well-known in this environment. (v. Beuther et al 1987). However, obtaining sufficient information on the two-point cross-spectral tensor to apply the proper orthogonal decomposition in the jet mixing layer (or any flow for that matter) is an ambitious and difficult task. Because of this an approach was devised that consisted of doing the experiment in several phases. This allowed for the opportunity to develop and test the experimental techniques, and to utilize the insight gained in each step to improve the experiment. Of particular importance were the effects of grid density, sampling rate, spectral convergence, and time record length which had to be ascertained from the first few phases before the final experiment was performed.

The first phase was described in detail by Glauser et al (1985, 1987) and involved using a rake of seven single wire hot-wire probes radially spanning the jet mixing layer at x/D=3. This arrangement yielded instantaneous streamwise velocity data as a function of radius in the jet mixing layer. From this data the streamwise cross-spectral tensor was computed and the integral eigenvalue problem solved using it. The second phase was performed to resolve questions about the spatial resolution in the first experiment. It involved using a rake of 13 single wire hot-wire probes across the same span as in the previous phase. Again this arrangement gave instantaneous streamwise velocity data as a function of radius in the jet mixing layer, only now at 13 positions. The third phase was described in detail by Glauser and George (1987a) and involved adding the azimuthal variation to the problem in phase 1, again using single wires to measure only the streamwise velocity fluctuations. The azimuthal correlations were realized in the following manner: one rake of seven hot-wire probes (same as in the first phase) was fixed at an arbitrarily chosen azimuthal position (since the flow was axisymmetric) while another rake of 7 hot-wire probes was moved azimuthally. In all, 16 azimuthal positions were measured over a span of 180 degrees.

The fourth phase is described in detail here, and involved measuring in addition to the streamwise velocity, the radial velocity component. In effect, the first and third phases were essentially repeated, only now including the radial velocity. This final phase utilized two rakes of probes with 4 cross-wires on each rake so that cross-spectral combinations from eight different radial stations were possible. In addition, the rakes were rotated relative to each other so that 25 azimuthal positions were measured over a span of 180 degrees. From this data $A_{ij}(r, r', k_1, m)$ has been calculated and the integral eigenvalue problem solved. The results are compared to those in the previous investigations, and are used to infer the character of the coherent structures in the jet mixing layer.

3.2. *The Facility*

The blower-driven facility for producing an isothermal, incompressible, axisymmetric air jet is shown schematically in Figure 1. The jet nozzle was of fifth-order polynomial design with a length-to-diameter ratio of unity, and follows a straight section containing both honeycomb and screens. The exit diameter of the contraction is 0.098m, corresponding to a contraction ratio of 10:1. The exit velocity could be varied continuously from 0.5 m/s to 40 m/s by using an inverter.

For the experiments reported herein, the exit velocity was 20 m/sec, corresponding to a Reynolds number based on exit diameter of 110,000. At these conditions, the boundary layer at the exit was turbulent with an approximate thickness of 0.0012 m (based on $U = 0.99U_e$), the mean velocity profile was flat to within 0.1 percent, and the turbulence intensity at the exit plane was 0.35 percent. The spectrum of the fluctuating velocity at the exit plane was smooth, and contained no spurious peaks. More details on the facility are included in Glauser (1987).

3.3. *Instrumentation and Calibration*

Fluctuating velocities were measured using hot-wire probes grouped together on 2 rakes with 4 cross-wires on each rake (v. Glauser 1987). The spacing between the individual probes was 0.43 inches, covering a span from $r/D = 0.13$ to $r/D = 0.90$ (both rakes were combined to span the entire radial measurement domain). Each sensor was 5 microns in diameter and had a sensing length of 1.2 mm giving a length-to-diameter ratio of 220. The wires, 3 mm in length, were made of tungsten with copper plating utilized on the inactive length that was soldered to the prongs. Each probe formed one arm of either a Dantec (DISA) 55M10 or 56C16 CTA standard bridge used in conjunction with a Dantec 55M01 or 56C01 Main Unit respectively. The individual bridges were each set to give a wire overheat ratio of 0.8. Either 5m or 20m long cables were incorporated in the probe arm of the particular bridge. The response of each system (including probes, prongs, printed circuit boards, card edge connector and cables) to a square wave test was tuned so that there was less than 10 percent overshoot. This allowed for stable operation over the bandwidth of interest (DC – 6kHz).

The bridge top voltage of each anemometer was connected to a 8-pole Bessel low pass anti-aliasing filter. The filters were phase-matched to within one degree. Each of the filtered anemometer signals was digitized using a 16 channel, 150 KHz maximum aggregate sampling rate, 15 bit, simultaneous sample and hold Phoenix A/D converter unit which was interfaced to a DEC PDP11/84 minicomputer. A DEC RA81 500 M byte disk drive and a DEC TU81 high speed tape drive were used for storage of the data.

The hot wires were calibrated in the same facility in which the experiments were performed. The velocity calibration range covered the entire range encountered in the experiment, and was typically 1 m/s - 25 m/s. A digital linearizing scheme detailed by George et al (1987) in which the velocity is expressed as the sum of powers of the voltages was implemented. A modified cosine law (Champagne and Sleicher 1967) was utilized to extract the radial and streamwise velocity components from the cross-wire data. Glauser (1987) contains more details on the calibration scheme and the instrumentation.

3.4. *Spectral Analysis Technique*

The time *direction* in the jet being studied is stationary so that the proper orthogonal decomposition reduces to the harmonic decomposition in this case. It is therefore usually more convenient to directly form the Fourier transform of the incoming data (via FFT) and compute the space- frequency cross-spectrum than to work with the space-time cross correlation.

The spectra and cross spectra (between different components of velocity and spatially separated velocity components) were computed using the following equation (v. Tan-atichat and George 1985)

$$\frac{\overline{\hat{u}_i \hat{u}_j^{\prime *}}}{T} = S_{ij}(\overline{x}, r, r', \vartheta, f) \tag{3.1}$$

where i and j denote vector components, S_{ij} is the cross spectral tensor (S_{ij} is the Fourier transform in time of R_{ij} not yet transformed over ϑ as described by equation 2.9) T is the record length and the overbar denotes an ensemble average. The prime emphasizes that the transformed signals can come from different spatial points.

These spectra and cross-spectra must be ensemble-averaged to reduce the variability. The constraints on minimum record length can be considerably more severe for cross-spectra than that for the spectrum because of phase differences introduced by the convection of disturbances between points. Hence, a significantly longer record may be required for cross-spectra than for spectra (v. Tan-atichat and George 1985). Since the experiments in the jet were performed at one streamwise location ($x/D = 3$) the phase differences across the shear layer and in the azimuthal direction are small, and it was possible to use a single record length for all separations. In brief, the process was as follows:

(*a*) Compute $\hat{u}_i$ and $\hat{u}_j^{'}$ using a FFT.

(*b*) Multiply $\hat{u}_i$ times the complex conjugate of $\hat{u}_j^{'}$ and divide by T to get the spectral or cross-spectral estimate.

(c) Ensemble average over many independent spectral estimates as suggested by equation 3.1 to obtain S_{ij}. This was accomplished by block averaging (typically 300 blocks). The rate of convergence of our spectral estimate is the same as for any ensemble average, i.e. $\epsilon \sim 1/N^{1/2}$ where N is the number of independent spectral estimates and ϵ is the variability of the spectral estimator (George 1978). A 20 percent bandwidth digital smoothing filter was then utilized to reduce the fluctuations even further. This filter did not affect the phase over the frequency band of interest in this experiment (v. Oppenheim et al 1983).

Each channel was low pass filtered at 800 Hz and sampled at a rate of 2 kHz. This allowed us to resolve approximately one decade into the $-5/3$ range of the spectrum. The number of samples taken per data block or record was 1024. It was determined from initial work using much longer records that a one-half second record length (a bandwidth of 2 Hz) was adequate. The time integral scale in this flow is approximately equal to 0.0013 sec. so that the number of integral scales per time record was large (typically greater than 50), thus insuring negligible window effect on the spectral measurements (v. Tan-atichat and George 1985, Glauser and George 1992).

3.5. *Effect of the Measurement Grid in ϑ*

The correlations in the azimuthal direction are periodic so that Fourier modes were used in this direction. This involved fitting a Fourier series in the azimuthal direction to the cross-spectra $S_{ij}(\overline{x}, r, r', \vartheta, f)$. A major concern with this procedure was the aliasing of the higher modes into the lower by virtue of the fact that it was not possible to satisfy a Nyquist criterion in space due to the size of the probes and the number that would have been required, nor were the results spatially filtered. Glauser and George (1992) discuss in detail the problem of spatial aliasing in this flow. They conclude that the lack of spatial resolution in ϑ can introduce aliasing into the modal decomposition which can seriously degrade the results at even the lowest mode numbers. They demonstrate that the presence and amount of spatial aliasing can be estimated from a Nyquist diagram and a knowledge of the turbulence (in this case the $m^{-5/3}$ range) and conclude that the effect of the spatial aliasing on mode numbers of 10 or less is negligible for the present experiment.

3.6. *The Effect of the Measurement Grid in r.*

In view of the preceding discussion of spatial aliasing on the decomposition into azimuthal Fourier modes, it is reasonable to suspect that the measurement grid in the radial direction might also be important. The maximum number of POD eigenfunctions which can be calculated from the decomposition is limited by the number of radial positions at which the measurements have been taken. A sampling theorem was proven by Glauser and George (1992) which states that if M eigenfunctions are required to represent the field, then at least M measurement locations are required.

Lumley (1970) showed that the number of eigenfunctions required to represent the integrated energy in the inhomogeneous direction was proportional to L/l where L is the lateral extent of the flow and l is the integral scale in that direction. For the axisymmetric jet mixing layer, $L \approx 0.2$ - $0.25x$ while $l \approx 0.07$ - $0.1x$, so that by Lumley's criterion three eigenfunctions should be sufficient for this experiment. This number was confirmed by Glauser and George (1992) who present a comparison between two different experiments, one with 7 wires (grid points) across the radial span and the other with 13. They find very little difference between the POD results in both cases with the first 3 modes containing 85 % of the kinetic energy, confirming Lumley's afore mentioned criterion.

What resolution is required or where the measurements should be taken must be established empirically at present although Glauser and George (1992) suggest a theoretical framework for making these decisions. In particular they argue that the kernel must be represented well enough to integrate it. This criterion was shown to be satisfied in this experiment by comparing the 7 and 13 wire results. In all of the measurements reported below, an eight point grid was used in the radial direction, thus exceeding the minimum number suggested.

3.7. *Application of the Continuity Equation*

Because of the assumptions of Taylor's hypothesis, homogeneity in x and a constant convection velocity for mapping f to k_1, it is desirable to validate the application of the continuity equation which is used to obtain the remaining terms in the correlation tensor. A simple check is to compare the azimuthal velocity moments, extracted from application of the continuity equation, to the direct measurements of the azimuthal velocity moments obtained by Hussain and Clark (1981). The mean square azimuthal velocity at a given radial position can be obtained from equation 2.13 by letting $i = j = 3$, setting $r = r'$, $\rho = \vartheta = 0$ and summing $A_{33}(r, r', m, k_1)$ over m and integrating over k_1. The mean square azimuthal velocity moments calculated this way are found to be within 5 percent of the measurements of Hussain and Clark (1981). For more details on the application of the continuity equation to this data base see Zheng (1991).

4. Analysis of the Flow Field

4.1. *Instantaneous Velocities, Velocity Moments and Spectra*

At the high Reynolds numbers of the flow studied here ($Re_D \sim 10^5$) flow visualization experiments show little evidence of the well organized structures present at lower Reynolds numbers. The reason for this is not that they are not present (as is evident from the conditional sampling experiments of Hussain and co-workers 1986), but that the flow is considerably more complex. This is due to the increased number of Fourier modes which arise from the enriched environment for non-linear interaction (from the advection terms of the Navier Stokes equations) at high Reynolds number. Arndt and George (1974) have shown how an increasing number of Fourier modes in stationary or homogeneous fields limits the visibility of organized structures (see also George 1988).

One can appreciate the complexity of this flow by examining Figure 2 where one block of instantaneous streamwise and radial velocities for eight radial locations are shown plotted as a function of time. The marked difference in the amplitude and frequency content of the signals is clear. The increasing intermittency toward the outer edges and the lack of any easily identifiable organized events are evidence that a statistical approach to turbulence, and to the search for coherent structures, is needed.

Figures 3 - 5 show, as a function of radius at $x/D = 3$, measurements of the mean velocity, $\overline{U}$, and the streamwise and cross-stream rms velocity fluctuations,u', v', all normalized by U_e, compared to the results of Hussain and Clark (1981). They used the local momentum thickness defined as

$$\theta_m = \int_0^\infty (\frac{\overline{U}}{U_e})(1 - \frac{\overline{U}}{U_e})dr \tag{4.1}$$

to check for similarity in the jet shear layer. (Note that while the equations of motion do not admit to fully self-preserving solutions, the results of Hussain and Clark 1981 clearly show that the flow has organized itself so that it is nearly self-preserving, consistent with the conjecture of George 1989 and the observations of Khwaja 1981). The value of θ_m in our experiment at $x/D = 3$ was found to be 1.37 cm, corresponding to $\theta_m/x = 0.046$.

From these figures it can be seen that the various quantities compare quite well with those of Hussain and Clark (1981).

Spectra of the streamwise and radial velocity fluctuations for eight radial positions in the jet shear layer ($r/D = 0.13 - r/D = 0.9$) at $x/D = 3$ are shown plotted in Figures 6 and 7. All of the spectra around the center of the mixing layer and on the low speed side of the shear layer have at least one full decade of a $k^{-5/3}$ range.

In and near the potential core ($r/D = 0.13$, $r/D = 0.24$, $r/D = 0.35$) all of the spectra have a maximum away from the origin. As one progresses away from the potential core toward the center of the mixing layer ($r/D = 0.46$) the radial velocity spectrum still exhibits this maximum away from the origin. The streamwise spectrum, however, now has its maximum at the origin. This highly peaked nature of the streamwise spectrum near the potential core and its evolution into a more simple rolloff toward the outer edge has been documented by others previously (see for example Khwaja 1981).

Just to the low speed side of the center of the mixing layer ($r/D = 0.57$) the streamwise spectrum has its maximum at the origin. The radial velocity spectrum, however, still exhibits a maximum away from the origin. This result is consistent with the results of Bevilaqua and Lykoudis (1977), Cimbala (1984), and Antonia et al (1987) who argue that, for turbulent free shear flows, the v velocity spectrum is preferred for estimating the wavelength of the organized motion because it exhibits a more discernible peak at the the average frequency of the organized motion. The St_D, defined as fD/U_e, calculated from the frequency corresponding to this maximum (i.e., $\simeq 90$ Hz) is $St_D \simeq 0.45$ which is consistent with the preferred mode discussed by Hussain (1983). At the remaining positions on the low speed side of the shear layer ($r/D = 0.68$, $r/D = 0.79$, $r/D = 0.90$) all of the spectra have their maximum at the origin.

4.2. *Results of the Azimuthal Decomposition*

In this section we examine how the azimuthal correlations breakdown into azimuthal modes. Note, here we present the azimuthal modal structure before application of the POD in the radial direction. The results are basically the same before and after but we feel that the data presented this way will be more useful to the community at large. These extensive azimuthal correlations have only been briefly discussed by Glauser and George (1987b). Sreenivasan (1984) examined temperature and streamwise velocity azimuthal correlations for several x/D positions but with limited radial extent. Hussain and Zaman (1980) examined the streamwise velocity azimuthal correlations for excited and unexcited jets at three x stations, $x/D = 1.5, 2.8$ and 4.13 at the radial locations, $r/D = 0.33, 0.40$ and 0.467. The streamwise velocity azimuthal correlations obtained here agree qualitatively with those obtained by Hussain and Zaman at $x/D = 2.8$ for the unexcited jet. No other measurements have been reported in the literature of the radial velocity and Reynolds stress azimuthal correlations.

Equation 2.13 was solved numerically for the complex coefficients which, for the azimuthal direction, consisted of performing a 48 point transform for each r, r' and wavenumber (v. Zheng 1991). If equation 2.13 with $\rho = 0$ is summed over wavenumber k_1 we can suppress the streamwise dependence. It becomes under these conditions

$$R_{ij}(r, r', \vartheta) = \sum_m B_{ij}(r, r', m) e^{im\vartheta} \tag{4.2}$$

B_{ij} is then defined in terms of R_{ij} as

$$B_{ij}(r, r', m) = 1/2\pi \int_0^{2\pi} R_{ij}(r, r', \vartheta) e^{im\vartheta} d\vartheta. \tag{4.3}$$

It is useful to examine in detail the azimuthal correlations for which $r = r'$ so that various B_{ij} and R_{ij} can be plotted for each radial position in the jet mixing layer. Selected examples of these are plotted in Figures 8 - 11. For all of the correlations and their spectral breakdown versus azimuthal modes the progression begins near the center of the jet at $r/d = 0.13$ and progresses out to $r/d = 0.9$ on the low speed side of the shear layer.

Figure 8(a) shows measurements of R_{11} at the 8 positions in the jet shear layer. Near the potential core at $r/D = 0.13$ and 0.24 there is clearly a strong correlation over the entire 180 deg. span. At the remaining positions in the shear layer, $r/D = 0.35 - 0.9$ the correlations fall off somewhat more rapidly than that seen at the positions closer to the potential core, indicating the presence of substantially smaller scale turbulence and there is good correlation out past 25 degrees (although negative) for all of them.

The corresponding azimuthal mode spectra B_{11} are shown plotted in Figure 8(b). The zeroth mode (axisymmetric mode) can be seen to dominate near the potential core for $r/D = 0.13 - 0.35$ indicating a strong ring-like structure in this region. It is interesting to note that at $r/D = 0.35$ modes 4, 5 and 6 also start to play a role. As we progress out further in the shear layer from $r/D = 0.46 - 0.9$ the azimuthal mode spectra are substantially more broadband than in the potential core region although there is a moderate peak around azimuthal modes 4, 5 and 6 for these 5 positions.

Figure 9(a) shows measurements of R_{22} at the same 8 radial positions as above. At $r/D = 0.13$ the radial velocity correlation is seen to go to zero at 90 deg. and then become negatively correlated. As we progress out further in the shear layer the correlations fall off more quickly and exhibit less and less of an integral scale.

The corresponding B_{22} spectra are shown plotted in Figure 9(b). The first mode can be seen to dominate at $r/d = 0.13$ which suggests that either the ring-like structures flap back and forth or that they are tilted slightly. This was also seen by Long and Arndt (1985) and Long et al (1993) from pressure measurements in an axisymmetric jet. As we proceed out towards the center of the shear layer, $r/D = 0.35$ and out, the zeroth mode comes into play along with the first mode as opposed to what was seen in the center region of the jet where only the first mode dominated. From the center of the shear layer region and on out the azimuthal mode spectra become more broad band and the higher modes have significant energy content, indicative of smaller scale structures. It is interesting to note that the radial velocity azimuthal mode number spectra B_{22} do not exhibit the distinct peaks around modes 4, 5 and 6 as seen in the streamwise velocity azimuthal mode number spectra B_{11}.

Figure 10(a) shows the R_{33} obtained from application of the continuity equation for $r/D = 0.13 - 0.71$. Note the strong correlation versus azimuthal angle and similar shape exhibited for all positions across the entire shear layer. These are the *azimuthal* velocity azimuthal correlations so this behavior is not unexpected. Also note how all of the correlations become negative beyond 90 degrees.

The corresponding B_{33} are shown plotted for the 8 radial positions in Figure 10(b). All of these spectra are dominated by mode 1 and have basically the same shape for all 8 radial positions. Note how there is no contribution to B_{33} from mode 0. This is a direct result of the symmetry conditions discussed earlier.

Figure 11(a) shows measurements of R_{12} at the same 8 radial positions. At $r/D = 0.13$ there is clearly a strong correlation over the entire 180 deg. span much the same as was seen in Figure 13 at the same position. At $r/D = 0.35$ and 0.46 the correlations fall off very fast indicating the predominance of the small scale structures at these positions just to the high speed side of the shear layer. In fact at $r/D = 0.35$ the integral scale is estimated to be less than 10 degrees. For $r/D = 0.57 - 0.9$, the low speed side of the shear layer, the correlations become stronger again and exhibit significant correlation for up to 40 degrees or so (again negative beyond about 20 degrees as was seen in the R_{11}). It is important to note this behavior of R_{12} as the mixing layer is traversed: the strong correlation near the potential core; the lack of correlation just to the high speed side of the shear layer; and the stronger correlation as we progress to the low speed side. Note that the normal components shown above did not exhibit this behavior, but rather, the correlation versus ϑ became less or stayed the same as the shear layer was traversed towards the low speed side.

Figure 11(b) shows the corresponding B_{12}. Near the center of the jet, $r/D = 0.13$ and 0.24, modes 0 and 1 clearly dominate and the higher modes contribute little to the spectrum. For $r/D = 0.35$ and 0.46, just to the high speed side of the shear layer center, the lower modes also contribute, although in a negative fashion, but the higher modes all make a significant contribution and the spectrum is more broadband. This is consistent with what was observed in the correlations at the same positions. All of the B_{12} spectra on the low speed side exhibit significant contributions from azimuthal modes 4, 5 and 6.

The preference for the fourth, fifth and sixth modes as seen in all of the above spectra except for the B_{33} is intriguing. A sixth-lobe preference (although weak) was seen by Sreenivasan (1984) at a position closer to the potential core at $x/D = 1$. A detailed stability analysis for vortex rings has been carried out by Widnall and Sullivan (1973). From this analysis the number of preferential lobes is seen to depend on the circulation of the vortex as well as on the ratio of the vortex core radius to the ring radius. Under certain combinations of the above conditions they noticed a sixth-lobe preference. Thus the sixth lobe noticed in this work is not inconsistent with the results from Widnall and Sullivan (1973).

In summary, the $\overline{uv}$ azimuthal correlations are dominated by low order modes (0-2) near the potential core, and become broadband as one proceeds toward the center of the mixing layer. Then, toward the outside of the shear layer, modes 4-6 play an increasingly important role, as can be seen in Figure 11(a). These observations are consistent with those of Hussain (1986), in that they indicate either that the incoherent turbulence is convected toward the center of the large structure, or is generated there.

5. Results from the Proper Orthogonal Decomposition

5.1. *Overview*

A block diagram is shown in Figure 12 which summarizes the various phases of the study. The diagram is set up with the most complete problem at the top and various sub problems below. The complete problem consists of a four-dimensional (3 space plus time) vector decomposition which requires measuring $A_{ij}(r, r', k_1, m, f)$. This quantity was not measured completely so only partial decompositions were possible. Since there were a number of these (and keeping straight which is being discussed can be a real challenge), the various partial decompositions that were performed are shown as 3 different paths in Figure 12: From left to right, the various scalar, partial vector and full vector decompositions respectively. The section or reference where each of these are discussed is stated in the particular block. The order of the presentation in the text begins at the bottom of each path in the block diagram, and proceeds upward. The first results (the lower-most blocks), already presented in Section 4, are the velocity moments and spectra which can be obtained from the various decompositions or computed directly from the raw data. In Section 5.2 a two-dimensional version, partial vector version of the POD (radius plus time) is presented. Since the instantaneous velocity is known as a function of radius and time, this two-dimensional version allows for an examination of the instantaneous structure as obtained from the POD (that is, a partial projection as indicated in equation 2.6 can be performed). We are not able to do this in three-dimensions with the existing data base because the instantaneous velocity was not measured simultaneously at all azimuthal locations. The three-dimensional (radius, azimuthal angle plus time) scalar and partial vector decompositions are not presented here because they are discussed in Glauser and George (1987a) and Glauser and George (1987b) respectively and many of the conclusions are similar to what is observed in the full vector versions. In Section 5.3, full vector decompositions are presented for three-dimensional (radius, azimuthal angle and the streamwise direction) and one-dimensional (radius) configurations. The one-dimensional decomposition is helpful since it allows us to examine the radial behavior of the eigenfunctions without having to arbitrarily select a wavenumber/azimuthal mode number combination at which to do so. The results from the three-dimensional full vector decomposition, when interpreted along with the azimuthal dependence of the various correlations and their breakdown versus azimuthal modes as discussed in section 4.2, give much insight into the behavior of coherent structures in the near field jet mixing layer.

5.2. *Two-Dimensional Application of the Proper Orthogonal Decomposition*

A version of the proper orthogonal decomposition, where there is no azimuthal or streamwise dependence, which utilizes the velocity cross-spectra, can be derived from equation 2.10 for a fixed streamwise location $\overline{x}$ by setting ϑ, the separation in θ, equal to 0. The kernel in this case, $S_{ij}(r, r', f, \overline{x})$, is defined as the Fourier Transform of $R_{ij}(r, r', \tau, \overline{x})$ again with $\vartheta = 0$. Using the measured values of the this cross spectrum, the reduced version of equation 2.10 can be solved numerically for the eigenvalues and eigenfunctions. The Fourier transform of the velocity can be reconstructed from these eigenfunctions by using an appropriate form of equation 2.6

$$\hat{u}_i(r, f, \overline{x}) = \sum_{n=1}^{N} \hat{a}^n(f, \overline{x}) \phi_i^{(n)}(r, f, \overline{x}) \tag{5.1}$$

where the random coefficients are defined in this case by

$$\hat{a}^n(f, \overline{x}) = \int \hat{u}_i(r, f, \overline{x}) \phi_i^{(n)*}(r, f, \overline{x}) r dr. \tag{5.2}$$

A scalar version of this application was presented by Glauser et al (1985, 1987). Here we will discuss a partial vector version of the two-dimensional application.

In this experiment $S_{ij}(r, r', f, \overline{x})$ (with $ij = 11$, 22 and 12) was obtained on a $8x8$ grid in r and r' (from $r/D = 0.13$ to $r/D = 0.90$) along with 512 points in f. (It should be noted that this data base is just a subset of the data base which includes the azimuthal variation, but here the azimuthal variation has been suppressed by setting $\vartheta = 0$.) Because this is a only a partial vector decomposition and there is a finite grid in r, the number of eigenvalues and corresponding eigenfunctions obtained in this case will be 2 times the number of grid points (Moin and Moser 1989) so that $N = 16$ in equation 5.1 The numerical approximation consists of replacing the integral of the modified version of equation 2.10 by a suitably chosen quadrature rule; the trapezoidal rule was chosen for its accuracy and simplicity. For more details on the numerical approximation see Glauser (1987) and Moin and Moser (1989).

The cross spectra at any radial position are given by summing the individual contributions to various spectra, $S_{ij}^q(r, r, f, \overline{x})$ with $r = r'$ from the proper orthogonal modes.

This is written as

$$S^q_{ij}(r,r,f,\overline{x}) = \sum_{n=1}^{q} \lambda^{(n)}(f)\phi_i^{(n)}(r,f,\overline{x})\phi_j^{(n)*}(r,f,\overline{x}). \tag{5.3}$$

As q goes to N where N is the total number of eigenvalues and eigenfunctions (equal to 2 time the number of grid points in r, because of the two spatial components, which is 8 in this case) the total particular spectrum should be represented.

Figure 13 shows the first 3 eigenspectra plotted as a function of frequency. In this case the eigenspectra represent the contribution to the kinetic energy spectra from the streamwise and radial velocity integrated across the shear layer. The first mode dominates and contains about 40 percent of the total energy, consistent with what was found in the scalar case as described by Glauser et al (1985, 1987). The component velocity spectra at $r/D = 0.46$, with $ij = 11, 22$ in Equation 5.3, are shown plotted in Figure 14 with the superposition of the results of Equation 5.3 for $q = 1$, 2 and 3. Note how the first mode contains a significant amount of the energy and how close the $q = 3$ case is to the original for these spectra. This rapid convergence is consistent with what was observed in the scalar problem.

It is also of interest to examine the reconstruction of the instantaneous velocities for this situation. Equations 5.1 and 5.2 are applied to a particular record and the results inverse Fourier transformed to obtain the instantaneous streamwise and radial velocity signals as a function of time. The same approach as applied to the spectra to study their convergence is applied to the instantaneous velocities as well. The original u, v velocity field for the time record being examined is shown plotted in Figure 15(a) viewed in a reference frame moving at 12 m/s. The contribution from the first mode is shown in Figure 15(b). The basic characteristics are recovered with only one mode but the fine detail is not captured. The contribution from the first 3 POD modes is shown in Figure 15(c). Note how close the result is to the original vector field.

These results demonstrate the proper orthogonal decomposition to be quite efficient at organizing data. Moreover, the instantaneous properties of the random signal have not been lost, but only organized into the appropriate modes. So efficient has the scheme been at organizing the energy that only a few terms were needed to almost completely represent the instantaneous signal. These results, first presented for a $7x7$ grid in r by Glauser et al (1985, 1987), gave early support for the idea that the proper orthogonal decomposition provides an excellent set of basis functions for use in a dynamical systems approach to turbulence as discussed by Aubry et al (1988), Deane et al (1991), Glauser et al (1991, 1992), Rajaee et al (1994) and summarized by Berkooz et al. (1993).

5.3. *Three-Dimensional Vector Application of the Proper Orthogonal Decomposition using Taylor's Hypothesis*

These results extend the work discussed in Section 5.2 to include the azimuthal and streamwise spatial variation in the jet. It should be noted that this experiment was performed at one streamwise point, $x/D = 3$, so as a first step Taylor's hypothesis was utilized to obtain the streamwise development so that there is no time dependence in this application. The streamwise and radial velocity measurements are used in conjunction with the continuity equation to obtain the other components of the tensor to be used in the integral eigenvalue problem as discussed in Section 2.2. This results in a full vector problem for the 3 spatial directions as given in Equation 2.14 because all of the velocity components have been accounted for. Using the measured values (and those obtained by application of equation 2.12) of $A_{ij}(r, r', k_1, m)$, equation 2.14 is solved numerically for the eigenvalues and eigenfunctions. In this case $A_{ij}(r, r', m, k_1)$ was obtained on a

$8x8$ grid in r and r' (from $r/D = 0.13 - r/D = 0.90$) along with 25 points in ϑ (from 0 to π because of the azimuthal symmetry in the jet) and 512 points in k_1. Because we have the full vector field, there are 3N ($3x8 = 24$) eigenvalues and eigenfunctions for each wavenumber mode number combination.

5.3.1. *Results of a One-Dimensional Vector Decomposition*

A one-dimensional version of the POD allows for the radial variation of the eigenfunctions to be examined in a straightforward manner without having to arbitrarily pick a wavenumber mode number combination at which to examine the radial behavior (cf. Moin and Moser 1989). The convergence of the various components of the kinetic energy and Reynolds stress can also be easily examined with this definition. The problem to be solved can be obtained by setting ρ and $\vartheta = 0$ in R_{ij}, as defined in equation 2.13, and using the result as the kernel in the integral eigenvalue problem (Note: This is equivalent to summing $A_{ij}(\overline{x}, r, r', k_1, m)$ over m and integrating over k_1). This results in

$$\int R_{ij}(r, r', \overline{x}) \phi_j^{(n)}(r', \overline{x}) r' dr' = \lambda^{(n)} \phi_i^{(n)}(r, \overline{x}), \quad i, j = 1, 2, 3 \tag{5.4}$$

where r and r' denote different radial positions in the mixing layer and $\overline{x}$ denotes the fixed value of $x/D = 3$.

The radial behavior of the first 3 POD eigenfunctions $\phi_i^{(n)}(r)$, weighted by $(\lambda^{(n)})^{1/2}$, are shown plotted in figures 16 and 17 for the streamwise and radial velocity respectively. The eigenfunctions for the azimuthal velocity exhibit similar behavior. Note how ϕ_1 and ϕ_2 have the same sign for all three eigenfunctions throughout the domain. This results in a positive contribution from each individual POD mode to the production of turbulence kinetic energy since the mean gradient is negative across the shear layer. Also note how the higher POD modes exhibit more zero crossings. Both of these results are consistent with the results of Moin and Moser (1989) and Chambers et al (1988) who note the closer resemblance to Fourier modes with increasing order. It is important to note that in the jet we cover the whole domain in our integration so that we need not be concerned with partial domains as discussed in Moin and Moser (1989).

As was the case in section 5.2 for spectra, the Reynolds stresses and kinetic energy at any radial position can be obtained by summing the individual contributions to the particular Reynold's stress, with $r = r'$, from the orthogonal eigenfunctions. This is given by

$$\overline{u_i u_j}^q = \sum_{n=1}^{q} \lambda^{(n)} \phi_i^{(n)}(r) \phi_j^{(n)*}(r). \tag{5.5}$$

As q goes to N, where N is the total number of eigenvalues and eigenfunctions, the total particular Reynolds stress or components of the kinetic energy should be represented. Figures 18 - 20 show plots of the mean square streamwise and radial velocity and the Reynolds stress as a function of radius with the contribution of one term (q=1) from the right hand side of Equation 5.5, with $i, j = 11$, $i, j = 22$ and $i, j = 12$, superimposed on it. The first proper orthogonal mode is seen to contribute approximately 40 percent to the mean square streamwise and radial velocities and 85 percent to the Reynolds stress. This result is consistent with those of Moin (1984). The results of equation 5.5 with $q = 24$ (the total number of POD modes) were compared to the original moments and found to be the same to within machine accuracy, indicating that the software used to extract the eigenfunctions is internally consistent.

It is interesting to note that the contribution to the mean square streamwise, radial and azimuthal velocities from the first proper orthogonal mode is only about 40 percent as compared to 85 percent for the Reynolds stress. Recall that Lumley (1967) originally argued that the first term in the expansion could be identified as the large eddy. On the other hand, Hussain (1983) argued that a coherent structure is characterized by high levels of coherent Reynolds stress, but not necessarily a high level of kinetic energy. He further argues that if an eddy is viewed as a proper orthogonal eigenmode, then a coherent structure is not an eddy. The resolution of these two points of view lies in the argument put forth by Glauser and George (1987b) (see also George 1988) that the eigenfunctions are simply the building blocks from which an eddy is comprised. As the *eddy* or *coherent structure* evolves, different eigenmodes will be dominant at different times and phases in its life cycle. This will be discussed in the next section.

5.3.2. *Results of the Three-Dimensional Vector Decomposition*

A three-dimensional vector inhomogeneous problem was solved using the proper orthogonal decomposition. For each mode number wavenumber combination equation 2.14 was solved numerically using the values of $A_{ij}(r, r', m, k_1)$ obtained from the measurements and application of the continuity equation.

The first three eigenspectra for azimuthal modes 0 through 3 are shown plotted in Figures 21 - 24. The first POD eigenspectrum for each azimuthal mode is seen to dominate (this was the case for the remaining modes also) indicating that one term may be adequate for the description of the large eddy in the inhomogeneous directions. This is consistent with the earlier work discussed in Section 5.2 where the first term was seen to dominate as well. Note, however, the clear difference in the amplitude and wavenumber dependence of the various eigenspectra for the azimuthal mode numbers presented. The dominant eigenspectra for the lower modes (0 - 2) exhibit a maximum away from the origin; the maximum occurring at progressively lower wavenumber as the azimuthal mode number increases. The dominant eigenspectra for the higher modes however, have their peak at the origin (although not shown, the basic structure of the POD eigenspectra for azimuthal modes 4 and higher are similar to those for azimuthal mode 3). This highly peaked nature of the eigenspectra for the lower order azimuthal modes and their evolution into a simple rolloff for the higher azimuthal modes is similar to the radial behavior of the velocity spectra with increasing radius discussed in Section 4.2. The relative amplitudes of the eigenspectra also vary significantly as a function of azimuthal mode number. The amplitude is greatest for mode 0 and progressively decreases until mode 2. The amplitude then continually increases until mode 5 after which it decays monotonically. These results are consistent with the results of the azimuthal decomposition discussed in Section 4.4.1. There, azimuthal modes 0 and 1 were seen to play an important role near the potential core and modes 4, 5 and 6 farther out radially in the mixing layer.

This wavenumber/azimuthal mode number interdependence can be seen more clearly by examining Figure 25 which is a plot of the dominant POD eigenvalue versus streamwise wavenumber and azimuthal mode number. The fairly complicated structure indicates that there may be an exchange of energy or an interaction between streamwise wavenumbers and azimuthal mode numbers. In particular, there is an apparent energy path (remember that the eigenvalues are energy integrated across the jet shear layer) between the lower modes and modes 4, 5, and 6. This would not be inconsistent with the secondary instability type phenomenon, manifested by streamwise vortex structures as discussed by Bernal and Roshko (1986). This point will be discussed in more detail in the section which follows. The peak in the wavenumber direction for $m = 0$ corresponds to the Strouhal frequency of the jet. The peak in the mode number direction near $k_1 = 0$ is at approximately mode 5.

In summary, the results of the three-dimensional vector decomposition are consistent with all of the earlier observations, with the following additional features:

- The inclusion of the azimuthal direction reveals substantial insight into the three-dimensional structure of the jet mixing layer not available from the various lower-dimensional problems presented. This conclusion is obvious from the results of the azimuthal decomposition (v. Section 4.2) which reveals the existence of a coherent ring-like structure dominated by the axisymmetric mode near the potential core, but with the fourth, fifth and sixth azimuthal modes playing a significant role from the center of the mixing layer and outward toward the low-speed side of the mixing layer. It is even more clearly manifested in the eigenspectra extracted for the three-dimensional case discussed above. The clear difference in the amplitude and wavenumber dependence of the various eigenspectra for the azimuthal mode numbers presented is in stark contrast to the lower-dimensional problems discussed. In the lower-dimensional results the eigenspectra exhibit simple role off behavior (cf. Figure 13), like the higher azimuthal mode number behavior of the eigenspectra for the three-dimensional case. This is probably a manifestation of the type of aliasing often observed when lower-dimensional spectra are obtained from the full 3-dimensional spectra (v. Tennekes and Lumley 1972). Therefore, the highly peaked nature of the eigenspectra for the lower order modes and their evolution into a more simple rolloff for the higher azimuthal modes can only be determined from the three-dimensional decomposition. These results clearly show the limitations of the lower-dimensional problems. They also indicate that experiments where only slices of the flow are examined (i.e., only one spatial grid point in a particular direction) are clearly aliased by the modes not resolved. This can be viewed, in the context of Section 3.5 (Effect of the Measurement Grid in ϑ), as, in effect, trying to resolve the ϑ direction with only one grid point, and should cause serious concern to all who infer structure information from a single azimuthal slice. The problem of spatial aliasing is discussed in some detail by Glauser and George (1992).
- The inclusion of the azimuthal velocity components of the correlation tensor through application of Taylor's hypothesis and the continuity equation results in a change in the eigenspectra. The basic shapes of the eigenspectra are quite similar, although the amplitudes differ (except for mode 0), those obtained utilizing continuity being the higher. This is as expected since the eigenspectra which are calculated from application of Taylor's hypothesis and continuity use the full tensor which includes the contribution from the azimuthal velocity (see Zheng 1991).

6. Summary and Conclusions: A Two-Ring Model for the Jet

A proper orthogonal decomposition has been used to extract eigenvectors from two-point velocity measurements in the mixing layer of a high Reynolds number axisymmetric jet. Cross-spectra were measured over an $8x8x48x512$ grid at one streamwise location, $x/D = 3$. Application of the proper orthogonal decomposition in the radial direction yields the lowest order eigenfunction which contains 40 percent of the turbulent kinetic energy and typically 85 percent of the various Reynolds stresses in the jet mixing layer. It was possible to almost completely reconstruct instantaneous signals of the radial and streamwise velocity in the jet mixing layer from these proper orthogonal eigenfunctions using only the first few modes.

Further decomposition in the azimuthal direction, utilizing the harmonic decomposition, reveals the existence of a coherent ring-like structure dominated by the axisymmetric mode near the potential core, but with the fourth, fifth and sixth azimuthal modes playing a significant role from the center of the mixing layer and outward toward the low-speed side of the mixing layer. The Reynolds stress azimuthal correlations and their breakdown into azimuthal modes show that the incoherent turbulence is concentrated in the center of the coherent structure, indicating that it has either been generated or advected there.

From the results of the three dimensional vector proper orthogonal decomposition and the azimuthal harmonic decomposition it is possible to suggest a low-dimensional model for the evolution of coherent structures in the jet mixing layer (Glauser and George 1987b). This life cycle begins with inception of rings of concentrated vorticity from an instability of the mean flow. A mutual interaction then occurs between two different rings, analogous to the first stages of the leapfrogging type phenomenon. A vortex instability arises from the further interaction of these two rings which finally results in a cascade of energy to smaller scales by vortex breakup and stretching. These four stages are shown in Figure 26 and described in detail below:

- *(i)* **Formation From Base Flow:** Vortex ring-like concentrations arise from an instability of the base flow, the induced velocities from vortices which have already formed providing the perturbation for those which follow. This feedback mechanism has been suggested by others as well (e.g. Hussain 1983).
- *(ii)* **Attempted Leapfrogging:** These rings then behave like the text-book examples of groups of inviscid rings. While multiple ring interactions may occur, the interaction between pairs of rings dominates, at least in the absence of forcing. In particular, a rearward vortex ring overtakes the vortex ring ahead of it, the rearward vortex being reduced in radius and the forward are being expanded by their mutual interaction analogous to the first stages of the leapfrogging type phenomena (v. Yamada and Matsui 1978).
- *(iii)* **Instability:** The rearward ring is stabilized by the reduction in its vorticity and radius, and the increase in its core area, thus the predominance of the 0th mode on the high speed side (the potential core region of the jet). The forward ring has its vorticity increased by stretching as it expands in radius. This narrowing of its core while the radius is expanding causes the vortex to become unstable, thus the predominance of azimuthal modes 4-6 from the center of the shear layer outwards. This is similar to the Widnall-Sullivan mechanism (v. Widnall and Sullivan 1973 and Yamada and Matsui 1978). The growing wavy deformation of the leading vortex ring causes it to acquire a streamwise component of vorticity which accelerates the instability. This is consistent with the description of Bernal and Roshko (1986), the numerical results of Martin and Meiburg (1991) and those of others (v. Hussain 1986, Liu 1989).
- *(iv)* **Breakdown and Entrainment:** The continued effect of the rearward vortex on the forward and the now highly distorted ring, accelerates the instability until its vorticity is now entirely in small scale motions, in effect an energy cascade from modes 4-6 all the way to dissipative scales. This incoherent turbulence is swept to the outside by the induced velocity from the rearward ring. Then some of it is entrained by the still-intact rearward vortex as it passes by. Some of it is even entrained back to the center of the mixing layer, thus explaining the broadband character of the Reynolds stress at positions near the center of the shear layer (v. Figure 11). This is consistent with Hussain (1986) who argues that the incoherent turbulence is advected to the centers of the coherent structures. In this context then, this collecting of the debris, both small-scale vorticity and fluid material, would be what has been recognized as “pairing”. This entrainment of small scale vorticity into the core of the surviving rearward vortex significantly amplifies the small scale vorticity by stretching, and increases the local dissipation rate there.

The entire process is then repeated as a new rearward vortex overtakes and destabilizes the one ahead of it. Recent work of Lim (1997) provides evidence of this type of two-ring interaction.

What is new in the above description is the sequencing of the events. The above sequence is consistent with the observations of this experimental work and others (v. Hussain 1986, Liu 1989 and II) and the recent numerical results of Martin and Meiburg (1991). In the absence of full field measurements (all positions simultaneously), the model put forth above must be viewed as only a hypothesis.

Glauser et al (1992) have recently developed a proper orthogonal decomposition based dynamical systems model for the jet shear layer using the eigenfunctions reported here which has given some insight into the temporal behavior of these events. They find clear evidence of pairs of vortices interacting in the streamwise direction resulting in a transfer of azimuthal to streamwise vorticity. These results are not inconsistent with the initial stages of what has been proposed here. They are not able to shed light on the later stages of the coherent structures life-cycle because of the *low-dimensional* nature of their simulations in which the small scales are not resolved but modeled.

Further confirmation of the model proposed here will require experiments of sufficient spatial and temporal resolution to verify the sequencing of the later stages of the life-cycle of the coherent structure. These experiments will provide the needed information to objectively extract the sequencing of events throughout the entire life-cycle of the coherent structure and thus will require measurements simultaneously at all locations. Such experiments, using hundreds of probes to sample the full flow field simultaneously, are presented in part 2 of this paper and lend credence to the afore-mentioned model.

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FIGURE CAPTIONS

Figure 1. The facility used for producing an isothermal, incompressible, axisymmetric air jet.

Figure 2. One block of instantaneous streamwise and radial velocities for eight radial locations, plotted as a function of time.

Figure 3. Mean velocity $\overline{U}$ normalized by $U_e = 21m/s$: ($*$), results of present study; (-), results of Hussain and Clark (1981).

Figure 4. RMS streamwise velocity u' normalized by $U_e = 21m/s$: ($*$), results of present study; (-), results of Hussain and Clark (1981).

Figure 5. RMS radial velocity v' normalized by $U_e = 21m/s$: ($*$), results of present study; (-), results of Hussain and Clark (1981).

Figure 6. Streamwise velocity spectra at $x/D = 3$: (a) $r/D = 0.13 - 0.46$, (b) $r/D = 0.57 - 0.90$.

Figure 7. Radial velocity spectra at $x/D = 3$: (a) $r/D = 0.13 - 0.46$, (b) $r/D = 0.57 - 0.90$.

Figure 8. (a) Streamwise velocity azimuthal correlations, R_{11}, (b) Streamwise velocity azimuthal mode number spectra, B_{11}, for 8 radial positions in the jet shear layer at $x/D = 3$.

Figure 9. (a) Radial velocity azimuthal correlations, R_{22}, (b) Radial velocity azimuthal mode number spectra, B_{22}, for 8 radial positions in the jet shear layer at $x/D = 3$.

Figure 10. (a) Azimuthal velocity azimuthal correlations, R_{33}, (b) Azimuthal velocity azimuthal mode number spectra, B_{33}, for 8 radial positions in the jet shear layer at $x/D = 3$.

Figure 11. (a) Reynolds stress azimuthal correlations, R_{12}, (b) Reynolds stress azimuthal mode number spectra, B_{12}, for 8 radial positions in the jet shear layer at $x/D = 3$.

Figure 12. Block diagram which summarizes various phases of the study.

Figure 13. First 3 eigenspectra from partial vector case.

Figure 14. Contributions from the first, first two and first 3 proper orthogonal modes to: (a) a streamwise velocity spectrum, (b) a radial velocity spectrum; at $r/D = 0.46$ for the partial vector problem.

Figure 15. (a) An original u, v velocity vector field, viewed in a frame of reference moving at $12m/s$. (b) Contribution from the first POD mode to the original u, v velocity vector field. Note the large scale features are similar to what is observed in (a) but the small scale features are lost. (c) Contribution from the first 3 POD modes to the original u, v velocity vector field. Note how most of the small scale features are recovered.

Figure 16. First 3 POD streamwise velocity eigenfunctions plotted as a function of r/D, weighted by $(\lambda^{(n)})^{1/2}$.

Figure 17. First 3 POD radial velocity eigenfunctions plotted as a function of r/D, weighted by $(\lambda^{(n)})^{1/2}$.

Figure 18. Mean square streamwise velocity plotted as a function of r/D with 1 POD mode contribution superimposed.

Figure 19. Mean square radial velocity plotted as a function of r/D with 1 POD mode contribution superimposed.

Figure 20. Reynolds stress plotted as a function of r/D with 1 POD mode contribution superimposed.

Figure 21. First 3 eigenspectra for azimuthal mode 0, plotted as a function of streamwise wavenumber.

Figure 22. First 3 eigenspectra for azimuthal mode 1, plotted as a function of streamwise wavenumber.

Figure 23. First 3 eigenspectra for azimuthal mode 2, plotted as a function of streamwise wavenumber.

Figure 24. First 3 eigenspectra for azimuthal mode 3, plotted as a function of streamwise wavenumber.

Figure 25. Dominant eigenspectra plotted as a function of streamwise wavenumber and azimuthal mode number.

Figure 26. Four stages of turbulence production from two-ring model.

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