FLOW STRUCTURE IDENTIFICATION FROM MULTI-POINT MEASUREMENTS

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Abstract

Some of the possibilities for inferring the structure of complicated flows from simultaneous measurements at many points are reviewed. Conditional sampling, pseudo flow visualization, stochastic estimation, and the proper orthogonal decomposition are briefly reviewed and illustrated by example. Resolution criteria for multi-point spatial arrays are proposed which minimize the possibilities for misinterpreting the data.

1 INTRODUCTION

One of the most important attributes of the fluid dynamicist is the ability to visualize the fluid motions in the problem of interest. Flow visualization by tagging fluid material has a long and rich history, and has stimulated scientist, engineer and artist alike. Many a student, not to mention experienced investigator, has been fascinated by the crisp and revealing motion pictures of the NSF Fluid Mechanics Film series [34]. More than one has wished that he had the capability of revealing the details of his own flow with simple dye, smoke or hydrogen bubble experiments. But alas! It is an unfortunate fact of nature that the same fluids whose mysterious motions we seek to unravel, conspire through molecular diffusion and other processes to invalidate these simple tools. Thus most have had to recognize the futility of such efforts in a large number of flow environments, and have had to settle for far, far less information about what the flow is really doing. (The photo collection by Van Dyke [44] is proof, however, that at least a few have succeeded.)

For the experimentalist, this has usually meant settling for measurements of the average properties of the flow at a relatively small number of points in it. These limited experimental capabilties have undoubtedly had a negative effect on the efforts of theoreticians, both because of the lack of possibility for comparison with experimental data and the failure to overturn theories which were incorrect. One need only witness the role being played by supercomputer simulations of flows (in combustion, for example) to realize what might have been had not the

experimentalist had to settle for so little. Happily, for the experimentalist at least, it will be some time before full Navier-Stokes simulations of most engineering problems will become possible.

But the CFD community has not been the only one to benefit from the technical advances in hardware. One of the opportunities presented by the advances in electronics and computers over the past two decades is the possibility of making many measurements at many points in the flow simultaneously. Experiments utilizing tens of probes together have become routine, and now experiments using hundreds of probes are in the planning or in progress. The tranducers vary from thin film gauges to hot-wires, from optical scanners to holographic interferometry; and the flow environments vary from gas turbines to trees (Yes, real trees with branches and leaves!), from boundary layers to combustion chambers. All have the common objective: to obtain by computer imaging and statistical means a picture of what is really happening.

It is not the intent of this paper to provide a review of the variety of efforts hinted at above. Nor is it the intent to review the multiplicity of transducers which can be used for multi-point measurements. Instead, the focus will be two-fold:

- First, to demonstrate how measurements at many points can be used to infer the structure of the flow,
- Second, to decide what constraints must be placed on the measurements to ensure that proper interpretation is possible.

The first objective will be approached by a brief review of four basic techniques for handling mult-point data; namely, conditional sampling, pseudo flow visualization, stochastic estimation, and the proper orthogonal decomposition. To appreciate the need for these one must understand the character of high Reynolds number, often turbulent, motions. As modern full Navier-Stokes computer simulations have made clear, knowing the data at many points in the flow does little in and of itself to make clear what is happening because of the chaotic nature of the flow. The key to understanding usually lies in what is done to the data to bring the underlying order (we hope it's order) to the foreground. The four techniques chosen for discussion have found wide utilization over the past decade, and all hold forth the promise of extensive application in the future. All four will be illustrated by examples from the literature. Although the examples chosen all involve applications of hot-wire anemometry, there is nothing intrinsic to the techniques under discussion which limits their application. Thus they can be applied to any multi-point transducer array in most environments.

The second objective will be to provide resolution criteria for multi-point flow measurements. This is necessary if the purpose of the experiment is not to simply facilitate taking large quantities of single point data, but rather to enable understanding the flow's spatial and temporal structure. This resolution objective will be accomplished by first reviewing the requirements for single point measurements, then extending these requirements to include the constraints on spatial arrays in various environments. The goal (only partially realized) is to place the theory of spatial sampling at the same level of understanding as that for the digital sampling of time series. Although it will be necessary and convenient to represent the signals by their Fourier and proper othogonal decompositions (for homogeneous and inhomogeneous flows respectively), the results are believed to have wide applicability to all kinds of multi-point measurements, especially when used in conjunction with the interpretation techniques described above.

2 TECHNIQUES FOR IDENTIFYING FLOW STRUCTURE

The utilization of rakes of probes with good spatial resolution presents the opportunity to examine the structure of turbulent flow fields in ways that would not otherwise be possible. In this section several approaches will be discussed. The first involves using the simultaneous velocities obtained from rakes of hot-wires in conjunction with conditional sampling techniques. The second, termed pseudo flow visualization (PFV), consists of using the simultaneous velocity measurements obtained with hot-wire rakes to generate instantaneous velocity profiles. From these profiles the spatial extent and characteristic frequencies of the large scale structures can be determined. The third and fourth approaches involve using the simultaneous velocities in conjunction with stochastic estimation and proper orthogonal decomposition techniques.

2.1 Conditional Measurements

Simultaneous measurement using rakes of hot-wire probes have been utilized in conjunction with conditional sampling techniques by numerous authors to examine the large scale features of turbulent flows. These investigations have helped shed new light on the turbulence structure in a variety of flows including wakes, jets, free shear layers and boundary layers. In the following paragraphs, several examples have been chosen as illustrative; no attempt has been made, however, to provide a comprehensive review of the numerous applications.

In one of the first extensive applications of rakes of hot-wires, Blackwelder and Kaplan [6] examined the wall structure of the turbulent boundary layer using conditional sampling techniques. They used the two different rakes shown in figures **??** and **??**, the first to examine the instantaneous variation of the streamwise velocity in the direction normal to the flat plate and the other to examine the instantaneous variation in the spanwise direction. They found that the instantaneous streamwise velocity measurements normal to the wall exhibited a high degree of coherence over a large area in the direction normal to the wall. They also found from the spanwise rake measurements that there was evidence of a large scale correlation in the spanwise direction farther out in the boundary layer ($y^+ = 15$), but no evidence of the streaks that are apparent (from flow visualization studies) in the lower regions of the boundary layer. From their conditional measurements they concluded that the normal velocity was directed outwards in regions of strong streamwise-momentum deficit, and inwards when the streamwise velocity exceeded its mean value.

Teitel and Antonia [40] used an array of cross-wire probes in a fully developed turbulent duct flow. The rakes were deployed in the plane of mean shear to examine the interaction between the opposite shear layers, provided *simultaneous* information on the turbulence characteristics of the shear layers on either side of the centerline. A significant finding of their work was that instantaneous quadrant-2 events (u negative and v positive) on one side of the centerline can almost reach the opposite wall. It should be noted that this information was obtainable only from a rake of wires. They then argued that the contribution from quadrant-2 events to the Reynolds shear stress is smaller in duct flow than in a boundary layer, reflecting the mutually inhibiting effect of the flow structures associated with the opposite shear layers. Antonia and his colleagues (v. Antonia et al [3]) also used a similar arrangement to

examine the turbulent far wake of a circular cylinder. They were able to identify, using an array of 8 cross-wires, structures which were both symmetric and antisymmetric about the centerline.

The spanwise structure in the two-dimensional mixing layer was examined by Browand and Troutt [7]. They used a rake of 12 hot-wires across the span of the wind tunnel. Computer visualizations of the instantaneous hot-wire outputs showed that the large scale structures extended across the wind tunnel and that there was some spanwise irregularity. They inferred from this that the spanwise irregularity is related to interactions between adjacent vortices.

Hussain and his colleagues at Houston (v. Hussain [24]) have developed a technique for the "eduction" of structure which requires a rake of cross-wires. From these rakes they record the instantaneous velocity traces. They then smooth these traces via short-time averaging and infer the time evolution of 'pseudo vorticity' contours in the plane of the sensors. (Note that actual vorticity measurement would have required substantially greater resolution than was possible in the experiment.) Others, such as Nagib and his coworkers [33] at IIT, have used rakes of wires to study transition in various flows.

2.2 Pseudo Flow Visualization

All of the examples cited above plotted simultaneous velocity traces to gain insight into turbulent flows. This has recently been extended into a technique called *pseudo-flow visualization* (PFV), first detailed by Delville et al. [11] who utilized hot-wire rakes with *high spatial resolution* to create a graphical representation of the instantaneous velocity interactions in a flow field. Since conventional flow visualization techniques break down at high Reynolds numbers due to turbulent diffusion, pseudo flow visualization methods provide an alternative technique for *visualizing* the flow field in such cases.

The PFV method can be illustrated by the recent studies of Ukeiley et al [43] who examined the turbulent flow in a lobed mixer, a device for increasing mixing by enhancing streamwise vorticity (see figure **??**). The rake of single component hot-wires shown in figure **??**, was used to collect a record of instantaneous streamwise velocity-time traces. The rake contained 15 probes, each having a 5 micron tungsten wire with a sensing length of 1 mm. The rake spanned across a full lobe width with a hot-wire separation distance of 2.7 mm. In spite of the scope of the experiment, it was possible to collect the data with a personal computer data acquisition system.

Traditional analysis of hot-wire data involves plotting the instantaneous velocities at one location in space. However, a rake of hot-wires allows instantaneous velocities across the spatial extent of the rake to be plotted at each sampled time interval, thereby permitting insight into the relationships among all the measurements as time evolves. Figure **??** demonstrates the comparison between the PFV and the more traditional way of plotting instantaneous velocities.

PFV plots were created at five positions across the center lobes at each of three locations, 50, 100 and 150 mm downstream of the lobed mixer (refer to figure **??**). Figures **??** - **??** display the results obtained at these spatial locations. At 50 mm downstream there is a discernible difference between the five visualizations. Positions 2 and 4 show minimal fluctuations and small gradients, while positions 1, 3 and 5 are strongly indicative of the shear regions created by the lobes. At 100 mm downstream, visualizations of positions 1, 3 and 5 indicate

further development of these shear regions, while positions 2 and 4 show the beginning development of interactions between the two streams. By 150 mm downstream there is almost no difference in the PFV patterns between all five positions. This last set of pseudo flow visualizations suggest an increase in turbulence mixing, presumably at the expense of the mean streamwise vortices which have been shown by Eckerle et al [12] to begin to decay in this downstream region. Figure **??** shows the comparison of the PFV to a spectral measurement at position 1, 50 mm downstream of this lobed mixer. The approximate number of structures counted in the PFV plot corresponds with the frequency of occurrence of these structures as determined from the spectrum to be approximately 700 Hz.

This method was also applied by Delville et al [11] to a study of the structures in a turbulent, plane mixing layer. From the instantaneous velocity profiles, detailed structures were observed in both the transverse and spanwise directions. They were then able to utilize an edge extraction scheme to find an intermittency function which corresponded to the passage of the structures. From these results they found the spanwise to streamwise wave-length ratio to be approximately 0.16.

It should be noted that the structures observed utilizing the PFV technique in both experiments would be much more difficult (if not impossible) to infer from real flow visualization techniques due to turbulent diffusion effects at the relatively high speeds used in this study. The Ukeiley et al [43] study demonstrates that the hot-wire rake based PFV technique provides a simple and effective means for determining how well devices such as the lobed mixer perform, and that it potentially has many applications in industry. The limitations of the PFV technique in its present form are spatial aliasing (discussed later) and its inability to capture 3-D instantaneous structure.

2.3 Stochastic Estimation

One of the more popular ideas in experimental turbulence at present is that of stochastic estimation which provides a means for quantifying large scale structures in turbulent flows (v. Adrian and Moin [2] and references therein). In brief, stochastic estimation uses knowledge of a field at one or more points together with its statistical properties to infer its 'typical' behavior at other locations. Only Linear Stochastic Estimation (LSE) will be discussed here since Tung and Adrian [41] have shown that little is to be gained by going to second order or higher. Application of this technique requires knowledge of the two-point correlation tensor which can be obtained with only two probes. The conditional eddy can be estimated using various averaged quantities (for example, the Reynolds stress) to provide the condition. The use of rakes of probes, however, provides many more possibilities for estimating the simultaneous velocity vector field using the instantaneous velocities at one or more points across the span.

A conditional average can be defined as

$$< g(u)|E >= expected \quad value \quad of \quad g(u) \tag{1}$$

given that the event E, the detector of the coherent structure, occurs. It should be noted that the overbar denotes an average here and throughout the paper. However, since the properties of these coherent structures are not known beforehand it is difficult to determine reliable unambiguous and unbiased detector events. Adrian [1] suggested choosing

$$g(u) = u(x') \tag{2}$$

and

$$E = c \leq u(x) < c + dc \tag{3}$$

which confines the velocity vector to a small window between c and $c + dc$ where c is any arbitrary vector. Symbolically

$$\tilde{u}(x') =< u(x')|u(x) > \tag{4}$$

which can be approximated, for the linear estimate, as

$$\tilde{u}_i(x') = A_{ij} u_j(x). \tag{5}$$

Values for the coefficients, A_{ij}, are chosen such that the mean square error is minimized as

$$e_i =< [\tilde{u}_i(x') - < u_i(x')|u(x) >]^2 > \tag{6}$$

for $i = 1, 2, 3$. This minimization requires that

$$\frac{\partial e_i}{\partial A_{ij}} = 0 \tag{7}$$

which leads to an equation of the form

$$< u_j(x) u_k(x) > A_{ik} =< u_j(x) u_i(x') > \tag{8}$$

where $< u_j(x) u_k(x) >$ is the Reynolds stress tensor and $< u_j(x) u_i(x') >$ is the two-point correlation tensor.

These ideas can be illustrated using the recent application of LSE to the axisymmetric jet mixing layer by Cole et al [9],[10]. This work differs from previous studies in that the *instantaneous* velocities at more than one position across the jet shear layer provided the events. The simultaneous velocity and two point correlation tensor data of Glauser and George [19] were utilized for this application. In their high Reynolds number experiment ($Re = 100000$), 8 cross wires, spanning radially across the jet mixing layer at $x/D = 3$, were used to simultaneously measure the streamwise and radial velocity. The rakes were constructed on printed circuit boards similar in design to those used by Nagib and his colleagues at IIT and are described in Glauser [21].

Expanding equation (8) results in the following system of equations for the u, v data of Glauser and George [19]

First System:

$$\begin{array}{cccc} < u^2 > & < uv > & A_{11} & < uu' > \\ < vu > & < v^2 > & A_{12} & < uv' > \end{array}$$

Second System:

$$\begin{array}{cccc} < u^2 > & < uv > & A_{21} & < vu' > \\ < vu > & < v^2 > & A_{22} & < vv' > \end{array}$$

The estimated velocity components are now obtained by expanding equation (5) which results in

$$\tilde{u} = A_{11} u + A_{12} v$$

$$\tilde{v} = A_{21} u + A_{22} v$$

After applying this technique to the data of Glauser and George [19], the matrices that arise for a **single** wire estimate are

First System:

$$\begin{array}{cccc} < u_{ref}^2 > & < u_{ref} v_{ref} > & A_{11w} & < u_{ref} u_w > \\ < v_{ref} u_{ref} > & < v_{ref}^2 > & A_{12w} & < u_{ref} v_w > \end{array}$$

Second System:

$$\begin{array}{cccc} < u_{ref}^2 > & < u_{ref} v_{ref} > & A_{11w} & < v_{ref} u_w > \\ < v_{ref} u_{ref} > & < v_{ref}^2 > & A_{12w} & < v_{ref} v_w > \end{array}$$

where ref is the reference wire number and w is the wire number (i.e., 1 - 8 in this case).

For a **two** wire estimate these matrices become:

First System:

$$\begin{array}{cccccc} < u_{ref_1}^2 > & < u_{ref_1} v_{ref_1} > & < u_{ref_1} u_{ref_2} > & < u_{ref_1} v_{ref_2} > & A_{11w}^{ref_1} & < u_{ref_1} u_w > \\ < v_{ref_1} u_{ref_1} > & < v_{ref_1}^2 > & < v_{ref_1} u_{ref_2} > & < v_{ref_1} v_{ref_2} > & A_{12w}^{ref_1} & < v_{ref_1} u_w > \\ < u_{ref_2} u_{ref_1} > & < u_{ref_2} v_{ref_1} > & < u_{ref_2}^2 > & < u_{ref_2} v_{ref_2} > & A_{11w}^{ref_2} & < u_{ref_2} u_w > \\ < v_{ref_2} u_{ref_1} > & < v_{ref_2} v_{ref_1} > & < v_{ref_2} u_{ref_2} > & < v_{ref_2}^2 > & A_{12w}^{ref_2} & < v_{ref_2} u_w > \end{array}$$

Second System:

$$\begin{array}{cccccc} < u_{ref_1}^2 > & < u_{ref_1} v_{ref_1} > & < u_{ref_1} u_{ref_2} > & < u_{ref_1} v_{ref_2} > & A_{21w}^{ref_1} & < u_{ref_1} v_w > \\ < v_{ref_1} u_{ref_1} > & < v_{ref_1}^2 > & < v_{ref_1} u_{ref_2} > & < v_{ref_1} v_{ref_2} > & A_{22w}^{ref_1} & < v_{ref_1} v_w > \\ < u_{ref_2} u_{ref_1} > & < u_{ref_2} v_{ref_1} > & < u_{ref_2}^2 > & < u_{ref_2} v_{ref_2} > & A_{21w}^{ref_2} & < u_{ref_2} v_w > \\ < v_{ref_2} u_{ref_1} > & < v_{ref_2} v_{ref_1} > & < v_{ref_2} u_{ref_2} > & < v_{ref_2}^2 > & A_{22w}^{ref_2} & < v_{ref_2} v_w > \end{array}$$

where ref_1 and ref_2 are reference wires 1 and 2 respectively. The estimates for the two wire reference case are then

$$\tilde{u}_w = A_{11w}^{ref_1} u_{ref_1} + A_{12w}^{ref_1} v_{ref_1} + A_{11w}^{ref_2} u_{ref_2} + A_{12w}^{ref_2} v_{ref_2} \qquad (9)$$

and

$$\tilde{v}_w = A_{21w}^{ref_1} u_{ref_1} + A_{22w}^{ref_1} v_{ref_1} + A_{21w}^{ref_2} u_{ref_2} + A_{22w}^{ref_2} v_{ref_2}. \qquad (10)$$

Without much trouble this system can easily be expanded to include estimates of all 8 wires. An obvious property of the 8 wire estimate is that the estimated velocities will be exactly the same as the actual velocities.

Cole et al [9], [10] have estimated the velocity field using various references positions for both single and multi-point estimates. A time record of the original velocity vectors at the 8 radial positions across the jet shear layer at $x/D = 3$ is shown plotted in figure **??**. Figures **??**, **??** and **??** show the single point estimated fields using wires 3,4 and 5 respectively as reference. These were constructed using the single point versions of equations 9 and 10. Note the large differences between each of these conditional estimates and how they differ from the original vector field. It is clear that a one point reconstruction **does not** do an adequate job of estimating the entire flow. However, the conditional estimates obtained using wires located just off either side of the centerline of the shear layer do a reasonable job in reconstructing the other wires located on the same side, but capture very little from the opposite side of the shear layer.

Since the one point conditional estimates clearly bias the eddy detected, multi-point estimates were used by Cole et al. [9],[10] to try

and capture a more representative instantaneous conditional eddy. Figures ?? and ?? show estimates obtained using reference wires 3 and 5 and wires 3,4 and 5 respectively. These were constructed using equations 9 and 10 for the two wire estimates and a three wire version for the three wire estimates. Note how much better these results compare to one another and to the original random velocity field shown in figure ??. From these results the authors conclude that a two point reconstruction (specifically wires 3 & 5) **does** an adequate job in reconstructing the entire flow and little is gained by going to a 3 point estimate. They also demonstrate that for computing conditional averages, the number and proper placement of the references probes is critical.

2.4 Proper Orthogonal Decomposition

Another popular idea at present for characterizing flow structures in turbulence is the so-called proper orthogonal decomposition (POD). Like stochastic estimation, the POD requires knowledge of the two-point correlation tensor. This be can be obtained from two point measurements alone, so that if one is only interested in reconstructing averaged quantities such as spectra then the two point measurements are sufficient. If, however, the full power of the POD is to be exploited to reconstruct the instantaneous decomposed velocity fields, then rakes of hot-wires must be used. As will be shown later, this latter possibility presents opportunities for using the PFV technique to visualize the results of the POD.

The POD results from the search for a deterministic field which has the largest mean-square projection on the velocity field (i.e. the structure which maximizes the energy, v. Lumley [29]). Maximizing the mean-square projection, leads to the following integral eigenvalue problem

$$\int\int\int\int R_{ij}(\vec{x},\vec{x}',t,t')\phi_j^{(n)}(\vec{x}',t')d\vec{x}'dt' = \lambda^{(n)}\phi_i^{(n)}(\vec{x},t). \qquad (11)$$

The kernel of equation 11 is the two-point velocity cross-correlation tensor, $R_{ij}(\vec{x},\vec{x}',t,t') =< u_i(\vec{x},t)u_j(\vec{x}',t') >$, and the summation of the eigenvalues is equal to the total energy. The integral equation has an infinite number of orthogonal solutions which can be used to reconstruct the original random velocity by the following equation

$$u_i(\vec{x},t) = \sum_{n=1}^{\infty} a_n\phi_i^{(n)}(\vec{x},t), \qquad (12)$$

where the coefficients, a_n, are random and uncorrelated, and must be determined for each realizaion of the flow by projecting the eigenfunctions on it.

If a direction (or time) is assumed to be statistically stationary, homogeneous or periodic, the POD reduces to the more familiar harmonic decomposition so that Fourier analysis is used in these directions (v. George [16]). Assuming the flow to be homogeneous in the streamwise direction, z, and stationary in time, equation 11 reduces to

$$\int \Phi_{ij}(x,x',y,y',f,k_1)\psi_j^{(n)}(x',y',f,k_1)dx'dy' = \lambda^{(n)}(k_1,f)\psi_i^{(n)}(x,y,f,k_1), \qquad (13)$$

where $\Phi_{ij}(x,x',y,y',f,k_1)$ is the Fourier transform of $R_{ij}(\vec{x},\vec{x}',t,t')$ in the stationary and homogeneous directions. The ψ's

are the frequency and wavenumber dependent eigenfunctions and x and y denote the remaining inhomogeneous directions (see figure **??**). Note that often it is convenient to treat an inhomogeneous, but slowly developing flow as if it were locally homogeneous in the streamwise direction, as in the lobed mixer experiment discussed earlier and below.

Simpler decompositions using only some of the variables can also be used. For example, Ukeiley et al [43] considered the reduced decomposition given by

$$\int \Phi_{11}(x,x',f)\psi_1^{(n)}(x',f)dx' = \lambda^{(n)}(f)\psi_1^{(n)}(x,f). \tag{14}$$

where Φ_{11} is the measured one-dimensional spectrum across the flow. In equation 14, only the spanwise direction, x, is decomposed through the use of POD. The vertical direction, y, is also statistically inhomogeneous in this flow, but is not considered in this preliminary examination. The streamwise velocity component which has been decomposed with Fourier analysis can be reproduced in Fourier space by

$$\hat{u}_1(x,f) = \sum_{n=1}^{\infty} a_n(f)\psi_1^{(n)}(x,f), \tag{15}$$

where the random coefficients $a_n(f)$ can be calculated for a single realization of the transformed field by

$$a_n(f) = \int \hat{u}_1(x,f)\psi_1^{(n)*}(x,f)dx. \tag{16}$$

This equation is derived using a similar process to that of a Fourier decomposition (i.e., multiplying equation 15 by ψ^*, where the $*$ denotes the complex conjugate, and integrating over the whole region).

The numerical approximation, detailed by Glauser et al [18], simply consists of replacing the integral in equation 14 by an appropriate quadrature rule (in this study a trapezoidal rule). $\Phi_{11}(x,x',f)$ is obtained from experimental measurements and utilized in equation 14 to obtain the eigenvalues and eigenfunctions. These eigenfunctions are then utilized to reconstruct the original Fourier transformed random velocity field. This streamwise velocity component in Fourier space can then be inverse transformed to obtain the reconstructed instantaneous velocity-time trace. It should be noted that the random coefficients *could not have been* calculated unless rakes of hot-wires were used. This is because the instantaneous velocity at all points, x, are needed simultaneously so that the integral in equation 16 can be computed.

Figure **??** illustrates pseudo flow visualization plots of reconstructed instantaneous signals for various proper orthogonal modes of the lobed mixer flow as reported by Ukeiley et al [43]. The contribution from the first eigenmode, displayed in figure **??**b, shows a good representation of the dominant structures shown in figure **??**a. The first three eigenmodes combined capture the *global* features seen in the pseudo flow visualization of the total streamwise velocity field as illustrated in figure **??**c. The summation of the first five modes, displayed in figure **??**d reproduces the general shape of the original plot at all locations, however the smaller scales are not completely captured. A summation of the first seven eigenmodes is needed to obtain a reconstruction which captures the small scales, as seen by comparing figures **??**a and **??**e.

The contributions from subsequent modes are negligible. One can argue from these results that the large scale features of this flow field can be adequately represented by using the first proper orthogonal mode alone. These results indicate that a low dimensional dynamical systems approach may be fruitful for this flow. [4], [20].

3 RESOLUTION REQUIREMENTS

3.1 Overview

This section discusses some of the unique problems encountered in the interpretation of multi-point measurements. It should be obvious that the aggregate of the measurements can be no better than each of them individually. Thus, regardless of whether the data are taken simultaneously at many points simply to expedite the collection of single point data or because additional information is sought on the spatial characteristics of the field, the spatial, temporal, and dynamic range requirements for each probe are the same as for single point measuring techniques. (Reference [17] summarizes these requirements for turbulence measurement.) In brief:

- The dynamic range of the probe, calibration and supporting instrumentation must cover the appropriate range of the signals encountered at each location. Note that *what* dynamic range is appropriate is very much a function of *what* information is to be sought; generally the greater the dependence on the higher moments of the signal, the greater the dynamical range required.

- The spatial resolution must be adequate so that the information removed by the averaging over the measurement area (or probe volume) is within acceptable limits. Again, what is acceptable spatial filtering is a function of the dependence of the information to be gleaned from the signal on the smallest dynamical scales of the flow. A general guideline is that the largest dimension of the measurement volume (or area) must be less than about half the smallest scale which must be resolved.

- The temporal resolution of the probe and supporting instrumentation must be sufficient to capture the highest frequencies (or transient events) which are of interest. Often these limits are imposed by the physical principles governing the measurement device.

Additional constraints on the data acquisition must be imposed if the data are to be digitized:

- All channels should be sampled simultaneously to avoid introducing phase errors. This is usually accomplished by sample and hold amplifiers on each channel. Alternatively, in some situations the phase differences are irrelevant for the intended purpose of the data, or can be corrected for after transformation to Fourier space.

- The resolution of the A/D converter must be sufficient to minimize the quantization noise to acceptable levels, what is acceptable depending on the information to be sought.

- The sampling rate of the A/D must be adequate to ensure that the sampled data can faithfully reproduce the desired information.

These last three criteria defy general guidelines because they are so strongly dependent on what information is desired from the recorded data. Quantization noise is generally white in character, and thus most adversely affects those quantities which are dependent on the lowest spectral levels of a signal. This most commonly occurs in turbulence at the highest frequencies (since the turbulence spectra drop off rapidly there), and can cause serious errors in any measurement which depends on the dissipative scales of the flow.

Sampling rate criteria are probably the most familiar of all the experiment design considerations, but also the most often misapplied. If (and only if!) it is important to retain the spectral character of the signal in the recorded data (as opposed to simply amplitude information), then the data must be sampled at a rate greater than twice that of the highest frequency present in the signal to avoid aliasing information from one frequency to another. This is the familiar Nyquist criterion and will be seen to have its counterpart in the spatial considerations discussed below. However, if the desired information can only be obtained by reconstructing the instantaneous signal from the digital data (as in many conditional sampling experiments), then the required sampling rate may be as much as $5 - 10$ times higher than the Nyquist criterion would indicate. This is because the reconstruction is carried out for a finite record length signal for which the Whitaker interpolation formula does not apply [39], and usually by less efficient reconstruction algorithms. On the other hand, there are many questions which can be asked about the data (like what are its statistical moments?) where aliasing is not a problem, and sampling rates substantially lower than the Nyquist rate can and should be used [15], [39].

Finally, there are two additional considerations which affect the length of record and the quantity of data, the first applying to spectral estimation (or other processes related to it) and the second to all estimates of randomly varying data. The latter in essence requires that sufficient independent estimates of any statistical quantity are available to ensure statistical convergence. The former has nothing to do with statistical considerations but arises from the fact that the Fourier transform of a finite record of a signal is actually the convolution of the signal transform with the transform of the record "window". If the record length is not much longer than the longest time scale of the signal, the "spectral leakage" due to the window will adversely affect the spectral character of the recorded signal. This can, of course, affect any inference from the data which depends on its spectral character. Both of these have been discussed in some detail in many places, v. [39], [15].

In order to exploit the full potential of multi-point measurement techniques for the exploration of the spatial (or spatial and temporal) character of the instantaneous fields, there are additional considerations which must be applied to the design of the spatial arrays. It will be assumed hereafter that all of the concerns addressed above for single point measurements have been satisfied, and attention will hereafter be focussed on the unique aspects of multi-point experimental design.

3.2 Periodic and Homogeneous Fields

Spatial resolution requirements for periodic or statistically homogeneous fields are most naturally discussed in terms of a Fourier decomposition which can be shown to provide an optimal representation [16]. It must be noted that it matters not whether or not it is the intent

of the experimenter to spatially Fourier decompose the results of the measurements. *Any* attempt to reconstruct the field or to analyze the measurements by techniques which depend on spatial variations within it depends implicitly on the Fourier coefficients which constitute it. Thus the multi-point measurements must be performed with sufficient spatial resolution and extent to faithfully capture them.

For periodic fields, the appropriate eigenfunctions are given by $exp(-im\theta)$ where $m = \pm 1, 2, \cdots$ and the corresponding Fourier coefficients by

$$a_m = \frac{1}{2\pi} \int_0^{2\pi} u(\theta) e^{-im\theta} d\theta \tag{17}$$

From these, the field can be reconstructed using

$$u(\theta) = \sum_{m=-\infty}^{\infty} a_m e^{im\theta} \tag{18}$$

The number of Fourier modes which can be obtained is, in practice, limited to half the number of points at which the measurements are taken plus one. (The factor of a half is because the Fourier coefficients are complex; the extra coefficient is $m = 0$ which is computed from the average of all the data points.) This is exactly the spatial counterpart of the representation of digitally sampled periodic signals by Fourier Series.

For statistically homogeneous fields, a Fourier decomposition is also appropriate, except that the *mode number*, m, is replaced by the *wavenumber*, k, which can take any value on the interval $(-\infty, +\infty)$. Because the field is (by definition) of infinite extent, the Fourier coefficient becomes the Fourier transform given by

$$\hat{u}(k) = \int_{-\infty}^{\infty} u(x) e^{-ikx} dx \tag{19}$$

The reconstructed field is now given by

$$u(x) = \int_{-\infty}^{\infty} \hat{u}(k) e^{ikx} dk \tag{20}$$

From an experimental point-of-view, the difference between the Fourier series and the Fourier transform vanishes since the measurement field is always of finite extent and the number of measurements is finite. The finite extent limits the lowest (or fundamental) wavenumber to $2\pi/L$ where L is the extent of the measurements, while the finite number of measurement locations restricts the number of independent Fourier coefficients to half the number of measurement points. These are usually evaluated at integer multiples of the fundamental. This is, of course, exactly analogous to the Fourier decomposition of statistically stationary temporally varying signals.

A consequence of discretely sampling the signal in space is that (like its counterpart in time series analysis) the information at one mode (or wavenumber) can be aliased into lower modes. This is most easily demonstrated for samples taken at equally spaced distance intervals. Suppose M modes are required to represent the signal and only N modes can be computed (from 2N measurement locations). Then when $M > N$, the information in the m^{th} mode for $m > N$ (but less than $2N$) appears in the calculated $(m - N)^{th}$ mode. If $m > 2N$ (but less than $3N$) it is aliased to the $(m - 2N)^{th}$ mode, and so forth. It is important to note that once the data is aliased, there is no way to unaliase it and the modal composition of the original signal is irretrievable.

If the spectral content (mean square Fourier coefficients) of the signal is varying monotonically and dropping rapidly as the mode number increases, the effects of aliasing may be negligible. This is often the case when measuring one-dimensional turbulence spectra (usually inferred from temporal spectra) which always have significant spectral content at very low wavenumber and often peak there. On the other hand, modal analysis of spatially sampled data can and often does lead to situations where the spectral content of the lowest modes can be small compared to that of the higher modes. When this occurs, even a relatively high number of resolved modes (compared to that of the peak) can lead to significant aliasing of the *lowest* modes.

Figure **??** from ref. [22] shows a modal decomposition of the mixing layer of an axisymmetric jet using 30 and 48 positions around the circle. Note the apparent modal content in the first few modes of the 30 position data which is reduced when the circle is resolved by 48 positions. It can be shown by arguing that the spectra fall off smoothly at the higher modes (in fact as $m^{-5/3}$) that the low mode number peak is largely due to aliasing. This is substantiated by the Nyquist diagram of figure **??** for the 30 position case which shows where the information above mode 16 is aliased. Clearly the physical processes inferred from the aliased data would be quite different from those actually present.

Aliasing in temporal data analysis can be minimized (and sometimes avoided entirely) by low-pass filtering *before digitizing* to remove the Fourier content at frequencies above half the sampling rate. The counterpart for spatial sampling would be *spatial low-pass filtering*. The concept if primitively applied would necessitate using many probes closely enough spaced to allow resolution of all the modes, then smoothing adjacent probe data samples to remove the highest modes or wavenumbers. This is not possible in most applications because of practical limitations on the number of probes and how closely they can be spaced. As a consequence, most experiments to-date have simply ignored the aliasing problem and hoped it wasn't there, sometimes with very misleading results!

Ironically, the spatial aliasing problem can be addressed in a straightforward manner by exploiting to advantage one of the principal limitations on single-point measurements, namely the unavoidable spatial filtering arising from the finite spatial extent of the probe. By making the probe dimensions *large* enough to span the distance between the measurement sites, the resulting spatial filtering removes the Fourier content of the modes which would have otherwise been aliased. Note that an analogous type of temporal filtering was implemented before the advent of modern high capture rate A/D converters by averaging the signal across the entire interval between sample times (Kristensen [25]).

A less satisfactory alternative than direct spatial filtering is to utilize the interrelation of spatial and temporal disturbances in many fields, especially turbulence. The temporal fluctuations in a convected spatial field are only in part due to the unsteadiness in the field ($\partial/\partial t$), the remaining part arising from the spatial variations in the field being swept by the probe ($U_c \partial/\partial x$) where U_c is the effective convection velocity. If the turbulence intensity is low, the convected spatial disturbances can dominate the unsteady signal. When this happens, there is a correlation between the frequency of the disturbance seen by the probe, say f, and the size of the disturbance, say λ. Thus it is possible to remove at least a portion of the spatial information smaller than a given wavelength, say λ_o, by temporally low-pass filtering the data above $f_o = U_c/\lambda_o$.

The finite extent of the measurement field imposes a spatial window on the data exactly as does the finite length of record in time. If the measurements are performed between $(-L/2, L/2)$ the spectral window

$$W(k) = L\frac{sin(\pi kL)}{\pi kL} \tag{21}$$

is convolved with the measurements. While this does not cause any change in the validity of the data itself (i.e., each individual data point), it does adversely affect the determination of which Fourier coefficients comprise it. The problem is exactly analogous to the window problems resulting from the finite record length of temporal signals. In general, the smaller the spatial extent of the measurements relative to the scale of the disturbances containing the energy and the more rapidly the spectrum falls off with wavenumber, the greater the adverse effects of the resulting spectral leakage. Note that if the spectrum peaks away from the origin, the leakage can be in both directions away from the peak. This is, of course, not a problem for periodic fields (like those which have axial symmetry) as long as the entire field is considered, but it is a problem for homogeneous or locally homogeneous fields where only a portion of the field can be considered.

3.3 Inhomogeneous Fields

The establishment of criteria to govern the spatial sampling of inhomogeneous fields is of great importance because most engineering flows are strongly inhomogeneous in one or more directions. As for the homogeneous or periodic flows discussed above, the problem of deciding how many probes, where they should be placed, what their spacing should be, and how much of the flow they should span is obviously crucial to the successful inference of flow structure from the measurements. These questions represent a substantial challenge, in part because, unlike homogeneous or periodic flows, there is no convenient and general counterpart to the analytical Fourier modes from which the conclusions of the previous sections were drawn. Clearly, ideas like aliasing and spectral leakage must have there counterpart in the sampling of inhomogeneous fields as well, even though they are not well-represented by Fourier modes.

In order to quantify the problems, it is necessary either to choose particular fields or to select a means of representing a variety of them. The proper orthogonal decomposition discussed in Section 2.3 is a natural candidate for this discussion, both because it is a general method applicable to all inhomogeneous flows (it reduces to the Fourier decomposition for homogeneous or periodic flows), and because it is of interest in its own right (as evidenced by the activities discussed earlier). Whether one is interested in the POD or not, because it provides an optimal representation of the flow (in terms of capturing its energy with the fewest number of terms) it is hard to imagine that any other way of looking at the spatial characteristics of the field would have less stringent requirements.

For the purpose of this discussion, attention will be focussed on the one-dimensional decomposition posed by

$$\int_{region} R(x,x')\phi^n(x')dx' = \lambda_n\phi^n(x) \tag{22}$$

where ϕ^n is the n^{th} eigenfunction, λ_n is the corresponding eigenvector, and $R(x,x')$ is the two-point correlation. given by

$$R(x,x') =< u(x)u(x') > \tag{23}$$

For real fields, both the eigenvalues and eigenfunctions are real, and the latter are orthogonal and can be chosen to be orthonormal. The decomposition is optimal in that the lowest order eigenvalue is the largest, the next one is next largest, and so forth, so that the representation requires the fewest number of terms of any decomposition to represent the field.

A random field can be reconstructed from the eigenfunctions using

$$u(x) = \sum_{n=1}^{\infty} a_n \phi^n(x) \tag{24}$$

where the coefficients, a_n are random, in general, and satisfy

$$< a_n a_m >= \lambda_n \delta_{mn}. \tag{25}$$

For a single realization of the random field, the coefficients are given by

$$a_n = \int_{region} u(x)\phi^n(x)dx. \tag{26}$$

It is easy to show that the two-point correlation can be recovered by

$$R(x,x') = \sum_{i=1}^{\infty} \lambda_n \phi^n(x)\phi^n(x') \tag{27}$$

If the objective of the multi-point measurements is to determine the eigenfunctions and eigenvalues from experimentally determined values of the two-point correlati on, $R(x,x')$, then the problem reduces to obtaining data which are sufficient to solve numerically the integral of equation 22. The accurate determination of $R(x,x')$ is, of course, also a problem, but of the type discussed in Section 3.1. The problem of interest here is the measurement grid: How many points are required and where should they be located? While the Nyquist and window criteria of the preceding section would certainly be adequate (since the field can be expanded in Fourier modes, although less optimally), there are probably substantially less stringent requirements which will suffice. There is really very little work which has been done to determine what these criteria might be, and experimenters have largely relied on intuition and empirical tests. The following paragraphs attempt to summarize current understanding and set forth at least the beginnings of a sampling theory for inhomogeneous flows. The two questions of quantity and location will be considered separately beginning with the former.

It is obvious that the number of measurement locations in any experiment must be finite. Therefore the integral equation of equation 22 must be approximated by the matrix equation

$$R_{ij}\phi_j = \lambda\phi_i \tag{28}$$

The subscripts, $i,j = 1,\cdots,N$ represent the measurement points, and R_{ij} represents the correlation computed at pairs of these locations; i.e.

$$R_{ij} =< u_i u_j > \tag{29}$$

(Note that in practice R_{ij} is replaced by a more complicated matrix using appropriate weighting factors to assure that it is symmetric [18].) The techniques for solving this equivalent matrix eigenvalue problem are well-documented in many places (v. references [18] and [32] for particularly relevant discussions). Of primary importance here is the fact

that at most N linearly independent solutions exist corresponding to the N eigenvalues. Thus the number of measurement locations determines the maximum number of eigenfunctions which can be obtained. To see how many measurement locations are required it is useful to hypothesize that a field is *mode-limited*, which is defined to mean that all modes above a given number, say M, are identically zero. It is obvious from the above that the field can be properly resolved only if $N \geq M$. If $N > M$, then the first M eigenvalues necessary to specify the field will be determined and the extra $N - M$ eigenvalues will be zero. On the other hand, if $N \leq M$, the eigenvalues determined will not correspond uniquely to those of the original field, but will have additional information from those eigenvalues for $M > N$ "leaked" into them. This can most easily be demonstrated by attempting to determine a 3-mode field using only 2 locations. The two eigenvalues which can be determined are functions of all three eigenvalues actually present. Figure **??** from Glauser and George [22] shows different results for a jet mixing layer using 7 and 13 wire configurations. The differences are in part due to the aliasing of unresolved modes in the 7 wire case, and in part from the better approximation to the integral of equation 22 for the 13 wire case.

Lumley [30] argues that the number of terms required to capture most of the energy is proportional to the spatial extent of the inhomogeneity divided by the integral scale of the field, i.e. L/l. In the Glauser/George experiment, this number is about 3-5, and corresponds closely to the number of terms which proved to be significant in their experiment. For boundary layer flows, the integral scale is a strong function of distance from the wall in the near wall region. Thus, if the entire flow domain is utilized, the number of required terms can be quite large as discussed below. On the other hand, subdomains can be utilized to reduce the domain — one eliminating the near wall region where the integral scale is small, and another including only the near wall region so that the domain is small. The result is that the required number of terms is smaller for each subdomain. Note that while this works for the capturing of the turbulence energy, it does not imply that the higher terms may not be important for other kinds of processing like conditional sampling.

The discussion above can be illustrated by the results of Chambers et al. [8] and Moin and Moser [32] who used numerical simulations to study how the domain over which the integral is computed affects the resulting eigenvalues. For the boundary layer type flows they investigated, integration over the entire domain yielded a substantially larger number of eigenfunctions than did integration over subdomains consisting of the near wall region only and a region comprising most of the flow excluding the near wall region. In both these situations, the inner-outer character of the flow lends itself naturally to this kind of splitting of the problem. Presumably a proper interpretation of the decomposition for each regime would seek influences of one region on the other, perhaps through the coupling of the pressure field between them. No such physical reasons are available for splitting the jet mixing layer flow of Glauser and George [22] where the largest scales dominate the entire flow. The important message from these results is that regardless of the domain chosen, it will likely influence greatly the modal character of the reconstructed field, and most probably any inferences drawn from it.

Usually one can not guarantee that modes above a certain number are not present unless they are removed by spatial filtering, perhaps of

the type discussed in the preceding section. In the absence of detailed knowledge about the eigenfunctions, it is not possible to state precisely how such filtering should be carried out. It has been noted, however, by a number of investigators (v. Chambers et al. [8]) and Moin and Moser [32]) that the higher the mode number, the more closely the eigenfunctions begin to resemble Fourier modes. Thus, the criteria for homogeneous (in this case, locally homogeneous) flows provide useful guidelines.

Even if the number of measurement locations and the spatial extent of the measurement field are sufficient to satisfy the concerns above, that alone does not ensure that the eigenvalues and eigenvectors determined from the matrix equation (equation 28) correspond to those of the original integral equation (equation 22). The most difficult question of all is to determine in advance (without already knowing the eigenfunctions) where the measurements should be taken to ensure the best (or even an adequate) approximation to the real integral eigenvalue problem of equation 22. Certainly it would seem reasonable to concentrate the measurement grid in regions where the kernel, $R(x, x')$, is changing most rapidly, if care is taken to weight the matrix elements so as to not artificially redistribute the energy. This is the same kind of logic that lies behind the logarithmic spacing often used in boundary layer studies.

Some further insight into where the measurement points should be located (as well as some justification for the qualitative argument above) can be obtained by examining Lumley's [30] method of calculation by successive approximation where the $(n+1)^{th}$ approximation, say $\psi^{(n+1)}$ is related to the n^{th}, $\psi^{(n)}$, approximation by

$$\int_{region} R(x, x')\psi^{(n)}(x')dx' = \psi^{(n+1)} \tag{30}$$

where the first eigenvalue is obtained from

$$\psi_1^{(n+1)}/\psi_1^{(n)} \rightarrow \lambda_1 \tag{31}$$

and the first eigenfunction from

$$\psi_1^{(n)}/\lambda_1 \rightarrow \phi_1 \int_{region} {\phi_1}^* {\psi_1^0}^* dx \tag{32}$$

If the first "guess" is taken as $\psi_1^0 = 1$, then it is easy to see that the measurement locations must be chosen so that the integral of $R(x, x')$ over x' is correctly computed. There is no reason, in general, to believe that the best choice of locations for estimating the eigenvalues at one value of x will be the best for all. In view of this and in the absence of other information, a uniformly spaced grid might be the best compromise. Also, in the absence of more specific criteria, the current approach of halving (or doubling) the resolution to see how the eigenvalues change is perhaps the only way to confirm that the choices of grid are correct.

While the considerations above may ensure the correct behavior of the lowest modes, the higher modes may still present problems. This can also be seen from Lumley's method where the $(n+1)^{th}$ mode is calculated from the kernel minus its reconstruction from the first n-modes; i.e.,

$$\int_{region} [R(x, x') - \sum_{k=1}^{n} \lambda_k \phi^k(x)\phi^k(x')]\psi_{(n+1)}^{(m)}(x')dx' = \psi_{(n+1)}^{(m+1)}(x) \tag{33}$$

It is obvious that the higher the mode number to be calculated, the more it depends on the cumulative errors in the lower modes. These conclusions do not depend on the method of successive approximations, but are general in that the errors will be most pronounced in the higher modes.

4 SUMMARY AND CONCLUSIONS

Simultaneous multi-point measurements have been shown (both here and elsewhere) to provide unique tools for aiding in the understanding of turbulence structure in ways not otherwise possible with single point measurements. Several examples have been briefly reviewed which demonstrate how such measurements have helped shed new light on the turbulence structure in wakes, jets, free shear layers, channel flow and boundary layers. The interpretation of the data obtained with the rakes has been enhanced by using conditional averages, pseudo-flow visualization, stochastic estimation and the proper orthogonal decomposition.

The problem of multi-point measurements has been shown to be much greater than simply designing and building impressive arrays of probes which do not by their presence change the flow. An attempt has been made to make clear that there are a number of serious questions which must be addressed before spatially sampled data can be used to infer spatial and temporal structure in the flow. An obvious constraint that must be met is that spatial, temporal and dynamic range requirements for *each* probe on the rake are the same as for single point measuring techniques. In order to exploit the full potential of multi-point techniques for extraction of the spatial character of the instantaneous fields, however, there are additional considerations which must be applied to their design. With the aid of Fourier analysis for periodic and statistically homogeneous fields, spatial aliasing and spectral windowing are shown to be concerns that must be dealt with, just as they are in digitial time series analysis. An idea is proposed for *spatial low-pass filtering* which involves exploiting to advantage the spatial filtering arising from the finite spatial extent of the probe. An attempt has also been made, using the proper orthogonal decomposition, to establish criteria which govern the spatial sampling of inhomogeneous fields since ideas like aliasing and spectral leakage have their counter part here as well. It is argued that the criteria established for homogeneous fields would certainly be adequate for inhomogeneous fields but these are probably more stringent than actually required.

It has been noted that Fourier techniques and the proper orthogonal decomposition were used both to establish general sampling criteria and because they are of interest in their own right. Thus, even if it is not the intent of the experimenter to utilize these techniques to decompose the measurements, *multi-point measurements must still be performed over a sufficient span and with sufficient spatial resolution (or appropriate spatial low-pass filtering applied) so as to avoid spatial aliasing and windowing effects.* This is especially true when inferences are to be drawn from gradients computed from the instantaneous signals (e.g. vorticity).

Although the ideas discussed here have been developed for turbulent velocity fields, the advantages of multi-point measurements are obvious for other variables such as pressure and temperature and the various kinds of flow situations in which they occur. While the examples used herein have been entirely drawn from experiments using rakes

of hot-wires, the techniques used to analyze the measurements and the resolution criteria governing them are applicable to all types of transducers. With the rapid advancement of modern optical techniques for flow measurement, there is good reason to believe that multi-point measurement techniques will find increasing application in both scientific and engineering investigations. If care is taken to address the concerns expressed above, the increased physical insight and understanding resulting from them should be substantial.

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