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# Technique for Rapid Friction Factor Fluid Characterization

William K. George Jr.,* George B. Gurney,† and Neil S. Berman‡
*Applied Research Laboratory, The Pennsylvania State University, State College, Pa.*

**A device for rapid friction fluid characterization is described. Operational criteria are proposed and application to dilute polymer solutions is demonstrated.**

## Introduction

IN studies of non-Newtonian fluids it is often useful to characterize the fluid by its friction factor, vs Reynolds number behavior. The standard pipe flow provides a convenient experimental means for obtaining this information, because it is easily constructed and only the straightforward measurements of pressure drop and mass flow rate are necessary. The acquisition of data for a large number of mass flow rates can, however, become quite tedious and time-consuming if these are acquired by repetitive experiments. The technique described provides a method for rapidly obtaining the desired data by continuous varying the mass flow rate while maintaining the turbulence in near equilibrium.

## Apparatus

A schematic of the apparatus is shown in Fig. 1. The entrance to the pipe flow is connected to a reservoir, which is maintained at constant head. The pipe exit is connected through a flexible coupling to a closed container. When the valve is opened, the flow through the pipe is initiated; the maximum flow rate is determined by the initial pressure difference between the reservoir and the receiving container. As the receiving container is filled, the air initially filling it is compressed, and the pressure rises. This increasing pressure reduces the pressure difference over the pipe and the flow rate is retarded. The flow stops when the pressure in the receiving container reaches that of the reservoir. Both the pressure drop through the pipe and the container weight are continuously monitored, thus providing the necessary data. Suitable averaging of the data is necessary to yield a smooth plot of flow rate vs pressure drop.

## Evaluation of Flow Parameter

If the first pressure tap is sufficiently distant from the pipe entrance (at least forty diameters), the pressure gradient $\partial p/\partial x$ is effectively constant along the pipe because the turbulance has evolved to an equilibrium state. It is easy to show from simple momentum balance[2] that wall shear stress $\tau_w$, is given by

$$\tau_w = \Delta p \ (D/4L) \tag{1}$$

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*Research Associate, Department of Mechanical Engineering; presently Assistant Professor, State University of New York at Buffalo.

†Assistant Professor.

‡Visiting Professor, Department of Chemical Engineering; presently Professor of Engineering, Arizona State University.

where $L$ is the distance over which $\Delta p$ is measured and $D$ is the pipe diameter. If $p$ is the fluid and $g$ is the gravitational acceleration, the volume flow rate $Q$ is given by

$$Q = (1/\rho g) \ (\mathrm{d}w/\mathrm{d}t) \tag{2}$$

where $w$ is the weight of the receiving container. (This equation is exact, because only the weight of the fluid in the container is changing with time.)

The friction coefficient is defined by

$$C_f = (\tau_w / \tfrac{1}{2}\rho U_m{}^2) \tag{3}$$

where $U_m$ is the volume flow rate, $Q$, divided by the pipe cross-sectional area; that is

$$U_m = (4Q/\pi D^2) \tag{4}$$

The pipe Reynolds number is given by

$$Re = (U_m D/\nu) \tag{5}$$

where $\nu$ is the kinematic viscosity of the fluid. Hence, in principle at least, we may easily generate the $C_f$ *vs* $Re$ curve or, equivalently, the $Q$ vs $\tau_w$ curve.

## Analysis of Equilibrium

The basic problem with such a system is that it is not in equilibrium; that is, the turbulence is continuously adjusting to the changing flow conditions, since the pressure gradient is a monotonic function of time. In particular, Eq. (4) is strictly valid only if $\partial \bar{p}/\partial x$ is constant. In the following paragraphs we will establish a criterion to ascertain that Eq. (4) is applicable; i.e., that the flow is effectively at equilibrium.

For turbulent flow the dynamics of the turbulence will dic-

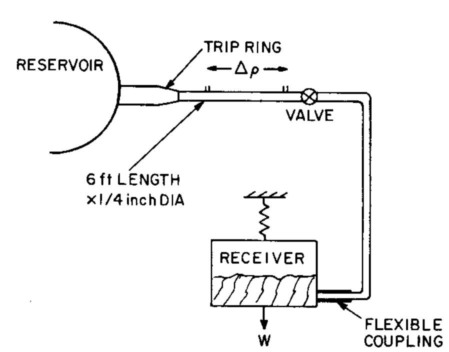


**Fig. 1 The apparatus. Pressure difference is created between reservoir and empty receiver. Flow is initiated by opening valve. Pipe pressure drop and receiver weight are continuously monitored.**

tate the flow development. To assure equilibrium of the mean flow, therefore, it need only be required that the turbulence is in equilibrium. This may be accomplished by requiring that the time scale for all external force changes be much greater than the largest time scale characteristic of the turbulence. The appropriate turbulence time scale is the time scale of the energy-containing-eddies, which is approximately[2]

$$T \cong (D/2u^*) \tag{6}$$

where $u_*$ is the friction velocity defined by

$$\rho u_*^2 = \tau_w \tag{7}$$

A time scale characteristic of the changing flow conditions can be taken as

$$T_{ext_1} = |Q/(dQ/dt)| \tag{8}$$

or alternatively, we could choose

$$T_{ext_2} = |\Delta p/(d(\Delta p)/dt| \tag{9}$$

For turbulent pipe flows an approximate relation between $Q$ and $\Delta p$ is given by

$$Q \sim (\Delta p)^n \tag{10}$$

where $n$ is typically around 1/2. Thus,

$$Q/\frac{dQ}{dt} \sim \frac{(\Delta p)^n}{n(\Delta p)^{n-1}(d\Delta p/dt)} = \frac{1}{n}\frac{\Delta p}{(d\Delta p/dt)} \tag{11}$$

and the two choices for $T_{ext}$ are simply related by a constant of order one. Hence, whichever $T_{ext}$ is used is immaterial.

The reduction of flow rate is caused by the compression of gas in the container. If the fluid being tested is a liquid, its high heat capacity effectively keeps the gas at constant temperature, since for all reasonable flow conditions $T_{ext}$ is much less than the time it takes the gas to reach equilibrium. (This is essentially a low Mach number approximation.) Hence, we treat the gas as if it were being compressed isothermally. Therefore,

$$pV_{gas} \cong \text{constant} \cong p_H V_f \tag{12}$$

where the constant has been evaluated at the final state, where $p_H$ is the reservoir pressure, and $V_f$ is the final gas volume.

In the experiments described later the isothermal assumption was observed to be true to within one part in three-hundred. It should be noted that because the deviations from isothermal compression are concentrated during the high flow rate part of the experiment, it is generally not possible to monitor flow rate by measuring container pressure.

Differentiating Eq. (12) we have

$$V_{gas}(dt/dt = -p(dV_{gas}/dt) = +pQ \tag{13}$$

because the change in gas volume is due to the increase of fluid volume. Hence,

$$(dp/dt) = (pQ/V_{gas}) \tag{14}$$

Because the reservoir is maintained at constant pressure

$$[d(\Delta p)/dt] = -(dp/dt) = -p(Q/V_{gas}) \tag{15}$$

Using Eqs. (9) and (15) our condition for equilibrium becomes

$$T_{ext} = |\Delta p/(\frac{d(\Delta p)}{dt})| = \frac{p_H - p}{p}(\frac{V_{gas}}{Q}) \gg T \tag{16}$$

where $p_H$ is the reservoir pressure and T is given by Eq. (6).

Using Eqs. (1) and (7) we have $u_* = (2c_f/2)^{1/2}\ Um$ which implies from Eq. (6) that

$$T \cong \frac{D}{2u_*} \cong \frac{1}{2C_f^{1/2}}\frac{D}{\bar{U}_m} = \frac{\pi}{4(2C_f)}\frac{D^3}{Q} \tag{17}$$

Using this in Eq. (16) we have

$$\frac{(p_H - p)}{p}\frac{V_{gas}}{Q} \gg \frac{\pi}{4(2C_f)^{1/2}}\frac{D^3}{Q}$$

or

$$\frac{(p_H - p)}{p}\frac{V_{gas}}{\frac{\pi D^3}{4(2C_f)^{1/2}}} \gg 1 \tag{18}$$

Clearly, the most serious problem will arise near the end of the run when $p$ approaches $p_H$. The best choice for $C_f$ is then a value near laminar-to-turbulent transition, where $C_f \sim 0.02$. Using Eq. (12), we may rewrite Eq. (18) as

$$(p_H - p)\frac{1}{p_H V_f}\frac{V_{gas}^2}{(5/4)\pi D^3} \gg 1 \tag{19}$$

Evaluating near the rest state where $V_{gas} \sim V_f$ and $p \to p_H$ we can write

$$\frac{p_H - p}{p_H}\frac{V_f}{(5/4)\pi D^3} \gg 1 \tag{20}$$

Hence, the flow can be maintained at near equilibrium arbitrarily close to the rest state (i.e., $p$ approaching $p_H$) by choosing $V_f$ large enough with respect to $(5/4)\pi D^3$.

If the receiving container is initially at atmospheric pressure $p_a$ the fluid volume can be expressed in terms of the initial volume $V_o$ and the reservoir pressure $p_H$ by

$$V_f = V_o(p_a/p_H) \tag{21}$$

and Eq. (20) becomes

$$\frac{p_H - p}{p_H}\frac{p_a}{p_H}\frac{V_o}{(5/4)\pi D^3} \gg 1 \tag{22}$$

Since $Q \approx (\Delta p)^{1/2}$ we can insure at least a decade of valid $Q$ variation by requiring that Eq. (22) be satisfied to at least

$$p_H - p \simeq 10^{-2}(p_H - p_a) \tag{23}$$

Substituting into (22), we have

$$10^{-2}(1 - \frac{p_a}{p_H})(\frac{p_a}{p_H})\frac{V_o}{(5/4)\pi D^3} \gg 1 \tag{24}$$

Taking $\gg$ to mean at least an order of magnitude, this condition becomes

$$(1 - \frac{p_a}{p_H})(\frac{p_a}{p_H})\frac{V_o}{(5/4)\pi D^3} > 10^3 \tag{25}$$

To achieve high flow rates, it is necessary to increase $p_H/p_a$ which can adversely affect the data at lower flow rates. To acquire data ranging over several decades in $Q$, it may be necessary to use the device twice, at high $p_H/p_a$ to get the high $Q$ data and at low $p_H/p_a$ to satisfy Eq. (25) for the low $Q$ data.

## Experimental Results

The facility has been illustrated in Fig. 1. The receiving chamber was suspended from a strain gage balance, and the

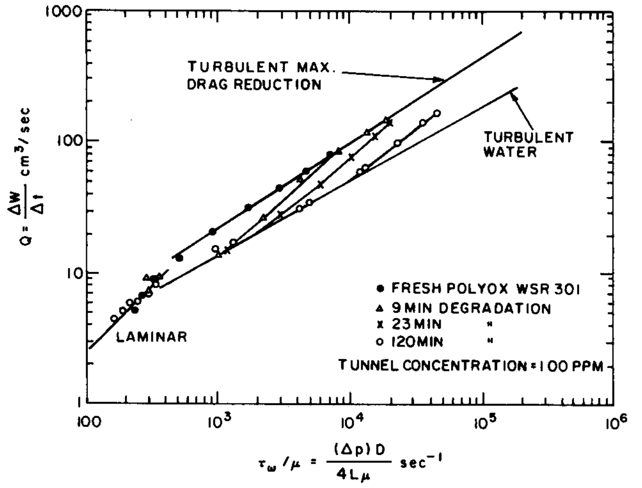


**Fig. 2 Data acquired from polymer solution, which was being continuously degraded by circulation in a 12-in. diameter water tunnel at 30 fps.**

pressure was monitored by a CEC ±25 psid transducer. Data were logged digitally, using an integrating voltmeter with a Hewlett-Packard data logging system, which alternately recorded pressure and weight. The weight was differentiated by simply differencing adjacent weight readings and dividing by the time interval, giving a flow rate point at the same time as the pressure drop point. To smooth noise fluctuations, it was necessary to use a one-second integration time for all data.

The over-pressure ratio $p_H/p_a$ used was as high as 2. The pipe diameter was 6.35 mm and the container volume was about 5 liters. For these conditions the equilibrium condition of Eq. (25) was satisfied.

Typical data for both water and polymer are shown in Fig.2. The polymer solutions were Polyox WSR 301 in concentrations of 100 ppm. Data shown are for different states of degradation. The data in Fig. 2 are shown as plots of $Q$ vs $\tau_w/\mu$ where $\mu$ is the viscosity. This type of plot was chosen because it displays the onset phenomenon of drag reducing polymers directly in terms of the appropriate flow strain rate $\tau_w\mu$. The solid lines indicate the well established data for turbulent and nonturbulent water and the maximum drag reduction asymptote of Ref. 3. The scatter evident for the lowest turbulent values of $Q$ represents the experimental manifestations of the nonequilibrium limitations (i.e., violation of the inequality of eq. (25); the scatter at the upper end of the laminar curve was due to intermittent behavior of the transition region and the sampling process used.

## Summary

A rapid technique for friction factor characterization of dilute polymer solutions and other fluids has been described. The limitations of the device have been explored, and a criterion for its use has been established. In view of the close relation between the drag-reducing friction factor curve at onset and the polymer time scale distribution, the technique could be used to great advantage in polymer test programs. The apparatus and operational procedures lend themselves well to on-line digital recording and analysis.

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