Designing Experiments to Test Closure Hypotheses

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Abstract

The unique problems confronting turbulence modelers and experimenters who wish to directly test closure hypotheses are reviewed. Particular attention is paid to the problems presented by systematic errors in the experimental data. Some of the sources of these errors in thermal and laser Doppler anemometry techniques for measurement are also reviewed, as are problems common to all techniques. The discussion is illustrated by examples from the authors' own work in axisymmetric jets.

Keywords

turbulence, modelling, experiment, Reynolds stress, closure, anemometer, hot-wire, laser Doppler

1 Introduction

In their pioneering book on turbulence modeling, Launder and Spalding [1] conclude a survey of the various approaches to modeling with strong words of caution to modelers and experimenters alike about the difficulties presented by experimental data which are wrong or misleading. Not only can countless hours be wasted and great expense be needlessly incurred, scientific progress and engineering advance can be stymied or side-tracked by the failed attempts and wrong-headed efforts to predict results which are not what they claim to be. The problem often occurs because the experimenters themselves do not understand or appreciate the detailed scrutiny to which their measurments will be subjected. It is compounded by the pressures from sponsors, supervisors, and self-imposed deadlines (like graduation) to produce results — often regardless of how hastily conceived, carelessly taken, or poorly processed. As a result, error correction is not infrequently the most important part of the data analysis. With the ever-present need to publish for survival or advancement, the increasingly severe page limitations which virtually prevent honest discussion of the results, and the casual attitude of many reviewers and editors who pounce on any *admitted problem* as an excuse for rejection, it is no small wonder that modelers find themselves frustrated by inconsistencies in the published data and feel no guilt in choosing their constants to match it instead of searching for the defects in the model itself. And in doing so lend credibility to the experimenter who covers his carelessness with the excuse, "Well, any measurement is better than nothing." As Launder and Spalding make clear, this is almost never true!

It is difficult enough without the pressures mentioned above to carry out any meaningful experiment in turbulence, and especially one which will be of value to turbulence modelers.

Not only must the experimenter contend with the statististical uncertainty of the data and the biases introduced by the measurement techniques, he must also have an understanding of how the boundary and initial conditions affect the flow field being measured. Generally, this requires ascertaining that the measurements satisfy the equations and boundary conditions *believed to govern the flow.* George [2] offers several examples of how the failure of turbulence models to accurately predict a flow and the discrepancies between various experiments could be explained by showing that the measurements did not satisfy the governing equations for the experiment *which was believed to have been performed.* Taulbee et al. [3] and Capp et al. [4] include the details for such an analysis of the axisymmetric jet, and argue that most of the problems in predicting this flow have been due to the failure of experiments to account for the effects of the ambient environment and boundary conditions on the flow being measured.

Thus the problem confronting both experimenter and modeler is often not that the experimental data are wrong, but that the experiment performed was not the experiment it was believed it to have been. George [5] suggests that any experiment is, in fact, *three* experiments:

- The experiment which was to have been performed.
- The experiment believed to have been performed.
- The experiment actually performed.

The objective of the experimentalist is to bring all three of these into coincidence. For the results to be useful for theoretical analyses, at least the last two must be the same; and this can be ascertained only by verifying that the measurements satisfy the governing equations and boundary conditions believed to be appropriate. It might be noted that this determination can not be made by simply corroborating the results by measuring the flow in several different ways, or even by showing agreement with previous experiments. Experiments similarly performed may still not be the ones believed to have been performed.

All of the problems above are compounded if it is the goal of the experiment to lend support to turbulence closure model evaluation. Moments of second and third order or higher are now of primary interest. These are, in general, more susceptible to technique-related biases which might be of acceptable order for lower order quantities. They are certainly far more sensitive to the manner in which the data are sampled, recorded and processed since they depend crucially on preserving the shape of the probability density function in the recorded data. Clipping, filtering, non-linearity in response, quantization errors, and noise can substantially alter moments of all orders, and especially those of odd order. Even the measurement of a second order quantity like the dissipation can challenge the limits of our experimental capabilities since attempts to measure it can be dominated by virtually every source of measurement error known today, even those which can usually be ignored (v. Hussein [6], Browne et al. [7])

Even when the difficulties outlined above can be overcome, the experimenter is usually still missing his most powerful tool, *the governing equations.* This is because it is often impossible to verify the measured results by showing that they satisfy the appropriate balance equations. The reason: generally one cannot measure *all* of the terms in the equations, those involving the pressure and dissipation representing the greatest hurdles. Thus these important moments are missing and must be inferred from the balance equations themselves, and cannot therefore be validated by the same equations. Even when by yeoman efforts the dissipation has been obtained directly, the terms involving pressure remain to be inferred.

The absence of the governing equations as a source of confidence in the measurements increases substantially the uncertainty of the results, and magnifies the importance of minimizing or eliminating the sources of bias and error so as not to mislead the theoreticians by the results. Ironically, it is often the models and closure relations themselves which must be used to validate the measurements whose very intent it was verify those same relations. This is like the proverbial "pulling one's self up by one's own bootstraps". However comforting this might be when theory and experiment appear to agree, it must always be recognized as never more than a poor substitute for direct experimental confirmation of theory.

This paper is about the difficulties of designing and carrying out experiments to test closure relations. It is important to note that one can seldom effectively test closure models by using them to predict particular flows, since often even poor models can be tailored to effectively "hindcast" a limited range of data. What is under discussion here is actually testing the closure assumptions at the level at which they are made. Thus, for example, if a particular model assumes a given relation between gradients of second-order moments and third-order moments, does the turbulence actually behave this way in the flow of interest or in general? It is only when these kinds of questions are addressed that turbulence modelers can say with confidence *where* the problems with the models are and *fix* them. Also, it is the only reasonable manner by which one can hope to have some idea in advance as to whether a particular model will work, and what level of model complexity is required. In brief, it is only this approach which can ultimately lead to improved engineering tools which can be applied with confidence.

If one doubts the need for this level of scrutiny of turbulence models, let him ponder for a moment the years that were spent seeking mixing length and eddy viscosity coefficients under the assumption that Reynolds stresses were proportional to velocity gradients. The modern age of turbulence can be said to have begun with the introduction of the hot-wire in x-configuration, the most elementary applications of which made it clear that such a simple assumption could not possibly describe many flows of interest. In spite of extensive efforts to provide a basis for closure assumption verification in a number of fundamental flows, our present turbulence models are in largely the same state as those of Prandtl and Taylor prior to the x-wire. We can *hope* there will be no major surprises as we explore the actual behavior of more complicated engineering flows, but the history of modeling successes and failures suggests strongly that there will be.

Given the limited objective of this paper, it is not particularly important which flows are to be considered since most of the difficulties in measurement are common to all. A distinction is made between measurements which are to be predicted by use of a turbulence model in the averaged Navier-Stokes equations, and measurements whose only objective is to verify the closure assumptions being used. This distinction is somewhat artificial since few can resist the temptation to try to calculate the entire flow once the closure approximations have been tested, and more than one set of objectively determined *constants* has been changed to make the calculation better fit the measurements. However, the distinction is important in *this* context since even measurements made in less than fully understood environments can be of value if made correctly. Thus, for example, while a set of measurments might be only a poor approximation to a jet in an infinite environment, they can still be of considerable value for the evaluation of local closure relations. Attempts to compute the overall flow will fail, however, because of the boundary conditions which must be assumed incorrectly to be homogeneous in the absence of information to the contrary.

In the following sections we shall try to illustrate by examples (largely drawn from our own work [1]) some of the methods by which closure hypotheses can be evaluated from experimental data. Attention will be largely focused on the moments which appear in the Reynolds stress equations, and on the difficulties in obtaining them by direct measurement or by inference from the averaged equations. Finally, a brief summary of some of the difficulties in making turbulence measurements with the more popular techniques will be provided, and where possible ideas for avoiding the problems will be given.

2 Evaluation of Closure Hypotheses with Experimental Data

2.1 Foreword

As noted above, the agreement of model predictions with measurements does not necessarily guarantee that the individual closure assumptions are correct. Incorrect formulations or parameters for two or more closures in the same model theory can compensate so that while predictions for the mean flow and even the turbulence kinetic energy may be essentially correct, other turbulence quantities inferred from the theory may be in error. The only sure way to validate turbulence models is to check each closure formulation with experimental data. Unfortunately, most data sets are not sufficiently accurate or complete enough to allow such validation. Because of these deficiencies the development of turbulence models has been somewhat hindered.

There are two main issues involved when using experimental data to validate closure for-

[1] One should not infer from this that we are the only ones involved in this kind of effort, as there have been a number of investigators carrying out experiments for the purpose of closure hypothesis evaluation over the past two decades or so.

mulations. The first is the lack of data for certain key quantities. All of the most widely used turbulence model theories involve the dissipation, and yet there exist very few measurements of this quantity. Also lacking are measurements of correlations between pressure and velocity. The second is the lack of accuracy of the measurements. Given the difficulty in measuring the dissipation and pressure - velocity correlations, such measurements of these quantities as do exist, are at best approximate. In addition, measurements of even the second and especially higher order velocity moments are often not of sufficient accuracy to enable curve fits to them to be differentiated for the evaluation of a closure hypothesis or for balancing a governing equation to obtain unknown correlations.

The purpose of this section is to present examples illustrating the use of experimental data in the direct evaluation of closure hypotheses. The difficulties in attempting to obtain certain correlations such as those involving pressure from equation balances are discussed, and the consequences of using inaccurate data in attempting to validate closures are also illustrated. A more detailed discussion of the considerations of this section from a somewhat different perspective is contained in Taulbee [8].

2.2 Gradient Hypothesis for Reynolds Stresses

Figure 1 illustrates a consistent set of data (Capp et al. [4]) for the Reynolds shear stress taken in an axisymmetric jet. Shown are curves for the $\overline{uv}$-profile measured with a burst-mode laser Doppler anemometer (LDA) and that for $\overline{uv}$ obtained from a balance of the axial momentum equation including the second order normal stress terms. The two curves are in good agreement, thus giving confidence in the measurement of $\overline{uv}$. Also, shown is a curve for the $\overline{uv}$ inferred from the mean momentum equation using only the mean velocity terms. The difference between this and the curve for the complete balance illustrates the importance of the so-called secondary terms and the extent to which an experimenter must go when possible to double check measurements. Figure 1 includes the Reynolds shear stress data measured with a hot wire. It can be seen that this curve is somewhat different than the LDA curve, the reasons for which are discussed elsewhere in this paper. The profile (not shown) for $\overline{uv}$ inferred from a balance of the momentum equation using hot wire data does not agree with the measured profile. Finally, Figure 1 illustrates the direct evaluation of the gradient hypothesis for the Reynolds shear stress $\overline{uv} = -\nu_t \partial U/\partial y$ with $\nu_t = 0.09k^2/\epsilon$ where k is the kinetic energy per unit mass and ϵ is its rate of dissipation. The curve was determined by evaluating the right side of the closure equation with LDA data and the dissipation obtained from a balance of the kinetic energy equation. (Note that a turbulence model was used for the missing pressure term, although it could have been neglected entirely with only a slight difference in the result, v. Taulbee et al. [3].) The model curve is in reasonable agreement with the measured curve from the LDA, as would be expected since the linear gradient model is known to work well for the shear component in near-parallel flows.

Figure 2 shows the normal Reynolds stress profile for $\overline{u^2}$ from LDA and stationary hot-wire HW measurements. This figure illustrates the sizable error in HW measurements in this moderately high intensity flow. Also shown is the gradient closure curve from $\overline{u^2} = -2\nu_t \partial U/\partial y + 2k/3$. The discrepancy between this and the LDA measured curve illustrates the inadequacy of the gradient hypothesis for the normal stress components. This inadequacy could easily be overlooked if the model were evaluated only on its ability to predict the mean flow, since the problems presented by it could have been absorbed in the model constants used for this particular flow. If the same model were used for other flows, however, the results might be seriously in error.

2.3 Dissipation

Fundamental to both the dynamics and the modeling of turbulence is the rate of dissipation of turbulence kinetic energy, or simply the dissipation. There have been very few attempts to directly measure the dissipation which depends on 12 different velocity derivative moments. (In fact, all 12 have never been measured.) Most experimental determinations of the dissipation depend on attempts to back it out of the kinetic energy equation by either ignoring the pressure velocity terms or modeling them, or by using some form of a local isotropy assumption with measurement of a single derivative moment. Taulbee et al. [3] discuss the efforts to apply these techniques to the jet data under discussion here.

Figure 3 shows the results of an attempt by Hussein [6] to directly measure various terms in the dissipation by using multiple wires and flying them through the jet. If the flow were truly locally isotropic, as is commonly assumed, all of the measured mean square derivatives should collapse to twice the mean square streamwise derivative. Clearly such is not the case, and local isotropy is not a reasonable assumption for even this relatively high Reynolds number flow. George and Hussein [9], however, argue from the same data that the flow is *locally axisymmetric* to a very good appproximation. (Note that the axisymmetry refered to here has nothing to do with the geometry of the jet, but rather the statistical symmetry of the small scale turbulence about a preferred direction.) Similar anisotropy has been noted by Browne et al. [7] for a turbulent cylinder wake, and their data is also very nearly locally axisymmetric (v. George and Hussein [9]). Figure 4 shows the dissipation profiles for the jet which have been calculated by assuming either local isotropy or local axisymmetry. Obviously these are quite different, and assumption of the former could lead to errors which could be quite significant. Also shown in the figure are the component dissipations which are nearly equally distributed among the three component equations. This is the expected result for isotropic turbulence, but also seems to be approximately true for locally axisymmetric turbulence, the non-isotropy of the individual derivatives notwithstanding (v. George and Hussein [9]).

In view of the importance of the dissipation profiles in the determination of the pressure velocity moments, it is worth noting that even the measurements above are subject to substantial errors, primarily arising from the uncertainties in the estimates of the cross-stream gradients. These errors could be as large as 20 %, and can radically alter both the magnitude and shape of the profile since the various terms do not contribute equally across the flow. This is easily seen from the expression for locally axisymmetric turbulence which is given by

$$\epsilon = \nu \left[\frac{5}{3} \overline{\left(\frac{\partial u}{\partial x} \right)^2} + 2 \overline{\left(\frac{\partial u}{\partial z} \right)^2} + 2 \overline{\left(\frac{\partial v}{\partial x} \right)^2} + \frac{8}{3} \overline{\left(\frac{\partial v}{\partial z} \right)^2} \right] \tag{1}$$

Thus, in spite of the best efforts to-date, and even with our new tool of local axisymmetry, it can not yet be said with confidence that a dissipation profile has been directly measured which is entirely satisfactory. [2]

[2] The derivative and dissipation measurements cited here have very recently been recognized to be based on an incorrect assumption about the probe used for some of the componenets. The revised measurements appear in George and Hussein [10] and appear to raise as many questions as they answer, and therefore have not been utilized here.

2.4 Triple Velocity Moments

The Reynolds stress model involves modeling the dynamic equation for the Reynolds stresses and, thus, formulating a closure for the triple velocity moment $\overline{u_i u_j u_k}$. One widely used closure formulation is that given by Hanjalic and Launder [11] as

$$\overline{u_i u_j u_k} = -C_s \frac{k}{\epsilon}\left[\overline{u_i u_l}\,\frac{\partial \overline{u_j u_k}}{\partial x_l} + \overline{u_j u_l}\frac{\partial \overline{u_i u_k}}{\partial x_l} + \overline{u_k u_l}\frac{\partial \overline{u_i u_j}}{\partial x_l}\right] \quad (2)$$

where in this case $C_s \approx 1/6C_1$ as given by Lumley [12] where $C_1 = 1.8$ is the return-to-isotropy coefficient. Shown in Figures 5 – 7 are triple moment profiles for the axisymmetric jet from LDA and HW measurements and those calculated from the gradient closure hypothesis using the LDA measured second moments.

Figure 5 shows that the $\overline{u^3}$ profile from the closure is in reasonable agreement with the hot wire measured profile. If it were not known that the hot wire measurements are seriously in error, one could easily draw false conclusions from them. However, an analysis of the LDA experiment and a reconfirmation of the LDA data by a flying hot wire experiment (Hussein et al. [13]) has shown that the stationary hot wire profile is in considerable error. Hence, similar to the gradient closure for the Reynolds stresses, the closure model for the triple moment does not work well for streamwise components.

Figures 6 and 7 show that the gradient closure model agrees reasonably well with the LDA measured profiles of the cross-stream components $\overline{vu^2}$ and $\overline{v^3}$. The hot wire measured profiles are, however, significantly different, a fact largely attributable to the cross-flow errors (see below). In fact for $\overline{vu^2}$ the shape of the hot-wire profile is grossly incorrect, completely missing the negative values near the centerline. This problem with the hot-wire triple moments had not been observed until recent LDA and flying hot wire measurements gave the correct shape for the profile. The shapes of the radial transport terms are very important in obtaining correct budgets from the turbulent kinetic energy equation. Note the closure model is consistent with the correct profile shape. Thus, the earlier measurements would have led to the erroneous conclusion that the theory was in error.

2.5 Pressure Diffusion

Many studies have appeared in the literature where experimental data has been used to balance the turbulence kinetic energy equation to obtain the dissipation. Without much proof it has been argued that the pressure diffusion is small and can be neglected. If the dissipation were known, then the pressure diffusion could be determined from the balance. Figure 8 shows the result for $\overline{pv}$ in the far region of an axisymmetric jet. In balancing the kinetic energy equation, the convection, transport and production terms were evaluated with fits to the LDA data and the dissipation taken from the flying hot wire measurements of Hussein and George. The pressure diffusion term $(1/r)\partial r(\overline{pv})/\partial r$ is a gradient term and should integrate to zero across the jet. The curve computed directly from the measurements shown in Figure 8 does not return to zero. It is likely (as was suggested above) that the problem is with systematic errors in the measured dissipation since the other terms involve only velocity moments which are believed to be given with reasonable accuracy by the LDA measurements. The second curve shown on the plot which does go to zero at large radius was calculated using a dissipation profile which was modified within the bounds of the 20 % error estimates on the cross-stream derivatives. Also shown is the pressure diffusion obtained from a model due to Lumley given by $\overline{pu_i} = -\rho\overline{u_i q^2}/5$. While the agreement with the model is better, one can take little comfort in either theory or data at this juncture. This is especially true for the latter which depend crucially on the dissipation estimate (see footnote above).

2.6 Pressure Strain-rate

In modeling the Reynolds stress equations, a closure must be found for the pressure-strain rate correlation, $\overline{p(\partial u_i/\partial x_j + \partial u_j/\partial x_i)}$. This term which partitions the turbulence energy among the components is very important in the Reynolds stress model theory, yet there are virtually no measurements with which to compare closure formulations. Backing the pressure-strain correlations out of the component Reynolds stress equations using experimental data is very difficult because of inaccuracies in the velocity moments and dissipation (see above). Because of the long infatuation with local isotropy (even though contradicted by most laboratory experiments) and in the absence of obvious alternatives, it is customary to absorb the anisotropy of the dissipation into the closure for the pressure-strain terms. (As noted above there is some support for this equi-partioning even in the absence of local isotropy.)

Figures 9 and 10 show the results of an attempt to obtain the radial and shear components of the pressure-strain correlation from the LDA data for the axisymmetric jet. Also shown are the results from the closure relation given by Launder et al. [14] as

$$\begin{aligned}\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)} &= C_1\epsilon\left(\overline{u_i u_j}/k - 2\delta_{ij}/3\right) - C_2\left(P_{ij} - 2P\delta_{ij}/3\right)\\ &\quad -C_3\left(D_{ij} - 2P\delta_{ij}/3\right) - C_4 k\left(\partial U_i/\partial x_j + \partial U_j/\partial x_i\right) \qquad (3)\end{aligned}$$

where $P_{ij} = -\overline{u_i u_l}\partial U_j/\partial x_l - \overline{u_j u_l}\partial U_i/\partial x_l$, $D_{ij} = -\overline{u_i u_l}\partial U_l/\partial x_j - \overline{u_j u_l}\partial U_l/\partial x_i$ and $P = P_{jj}/2$. The coefficients are $C_2 = (C_2{}' + 8)/11$, $C_3 = (8C_2{}' - 2)/11$ and $C_4 = (30C_2{}' - 2)/55$ with $C_1 = 1.7$ and $C_2{}' = 0.5$. These particular choices were shown by Taulbee [8] to be the best values for fitting the closure model to homogeneous shear flow experiments. As can be seen in the figures, the model and the experimentally inferred pressure strain terms agree very well for the shear component, but not nearly as well for the radial component. Because of the difficulties noted above for the dissipation, no definitive conclusions can be drawn from these balances of the Reynolds stress equations.

3 Experimental Techniques

3.1 The choice of a technique

As the preceding examples make abundantly clear, there is probably no single measurement technique which can provide all of the information desired. Each has unique strengths and weaknesses which may suit it for or hinder its performance in a particular environment. In the following sections, an attempt will be made to summarize the most important of the limitations of thermal and laser Doppler anemometer techniques. There are numerous other possibilities for flow measurement, and for most of these the limitations are well understood by at least a few.

Regardless of the particular technique, there are some general considerations which are applicable in evaluating the suitability of the technique for a particular measurement. Two of the most important of these are:

- Finite temporal response (or equivalently, finite bandwidth) – the failure of the sensor to respond quickly enough to the changing flow conditions.
- Finite spatial resolution – averaging of spatial variations in the flow over the sensitive surface of the transducer.

These effects can dominate attempts to measure the dissipation which depends heavily on the smallest scales of motion, and these therefore must be properly resolved if the results are not to be seriously in error.

Additional problems in measurement often arise because the vector quantities being measured (like velocity) are mapped by the transducer into a scalar output. It is this problem which is at the root of the problems with the hot-wire anemometer in high intensity turbulent flows, and which accounts for the large differences between the LDA and hot-wire results in the jet measurements discussed above. Whether or not these effects can be ignored at relatively low turbulence intensities is entirely determined by the magnitude of the quantity being measured *relative to the magnitude of the errors.* For example, the measurement of $\overline{vu^2}$ near the jet axis is dominated by the hot-wire cross-flow errors since the moment itself is very nearly zero. The effect can also be seen in the measured second moments at large radius where the cross-flow errors are becoming larger while the intensities themselves are diminishing. The LDA is immune to these particular effects (assuming sufficient frequency shift) since it responds to only a single component of the velocity.

Another problem closely related to that above is the problem of spectral aliasing which results when a spatially varying field (two or three-dimensions) is mapped into a temporally varying field (one-dimension). This occurs, for example, when a spatially varying field is convected over a stationary probe yielding time variations given by $\partial/\partial t = -\vec{u}\cdot\nabla$ (or equivalently, $\omega = \vec{k}\cdot\vec{u}$). Such a mapping is the basis for the use of Taylor's frozen field hypothesis which is often utilized (with other assumptions like local isotropy) for determination of the dissipation. Figures 11 and 12 from George et al. [15] illustrate the problems that can arise with the use of Taylor's hypothesis in the round jet, and shows the dependency of the errors on the turbulence intensity. These problems are independent of the type of transducer used, and can be usually be overcome only by moving the probes to reduce the effective turbulence intensity. Another alternative would be to directly measure the streamwise velocity gradients so that Taylor's hypothesis need not be invoked. Techniques to accomplish this with resolution sufficient for the dissipative scales have not yet evolved to practical tools for measurement.

3.2 Thermal anemometry techniques

Thermal anemometry techniques encompass two primary categories of transducers: hot-wires and hot-films. The basic principle behind both is the variation of their resistance with temperature, and when used with only a very small current (very low overheat ratio) both can be used as resistance thermometers. When provided with enough current to heat them significantly above the temperature of the surrounding fluid, the convective heat loss to the fluid can be related to the velocity of the fluid. There are numerous references describing these techniques in detail (e.g. Sanborn [16], Lomas [17]) and even more numerous review articles describing different aspects of their use in the measurement of turbulence (e.g. Corrsin [18], Comte-Bellot [19], and Blackwelder [20]). Our purpose here is to briefly touch on some of the most important limitations of thermal anemometry techniques which can not be overlooked in applications.

Foremost among the limitations of thermal anemometry are those arising from the vector to scalar mapping mentioned above. Since the heat transfer from most common configurations can achieve a specific value for a variety of flow directions, the ability to distinguish direction is lost. For example, a wire can not distinguish any of the components of velocity in a plane perpendicular to it, thus giving rise to the so-called *cross-flow* and *rectification* errors. It is unfortunately also weakly sensitive to the component of velocity along it, so that attempts to utilize its primarily normal cooling for directional determination are contaminated by the "k-factor" component (after the so-called k-factor in a modified cosine cooling law). Tutu and Chevray [21] and Beuther et al. [22] provide useful reviews of these phenomena. The latter also discuss some of the unique problems which can occur at low velocities because of the buoyancy introduced by the wire itself.

All of these effects become progressively more important as the turbulence intensity increases. It is especially important to note that the acceptable levels of turbulence intensity decrease as the order of the moment of interest increases. Thus, for example, while the errors in the measurements of mean velocity and turbulence intensity components near the centerline of a jet ($u'/U \approx 30\%$) are acceptable, the measurements of third moments at the same location are seriously in error. All of the measurements are hopelessly contaminated when the turbulence intensity is above 50%, or beyond the point at which the velocity is about half its centerline value. Such levels of turbulence intensity are quite common in many shear flows and close to walls, and are an important feature of most engineering flows. Thus the use of thermal anemometry techniques as stationary probes in such environments is not likely to yield much information of value, at least for the purposes under discussion here.

In spite of their limitations to low turbulence intensity environments, hot-wire anemometry techniques have unique advantages for the measurement of dissipation. They alone, at present, can provide the combination of frequency response, spatial resolution, and low noise necessary for the measurement of the smallest scales of the turbulence. They can be used in a variety of parallel and combination probes to directly measure the velocity gradients. Recent applications for this purpose include Browne et al. [7] and George and Hussein [10], the latter introducing the concept of *local axisymmetry* mentioned above. Note that neither of these sets of measurements used assumptions of local isotropy (and in fact showed that it was not an acceptable model of the flow), and both were careful to avoid the problems with Taylor's frozen field hypothesis described above, the latter by flying the hot-wires to reduce the *local* turbulence intensity.

Hot-film probes are, in general, less effective than hot-wires for several reasons. In addition to having even more complicated directional problems than hot-wires, their frequency response tends to be limited because of the roll-off introduced by the heat loss to the backing material. Also, it is not uncommon to encounter situations where the static calibration does not correspond to the dynamic calibration because of heat loss directly to the flow via the backing material. In spite of this, split films which use multiple film sensors on a single fiber are in common use. These configurations can have additional problems because of thermal cross-talk between the various films present (v. Herzog and Lumley [23]). None of these problems should be taken to imply that film probes can not be successfully used in a particular environment, but there is reasonable cause for skepticism about the results obtained if at least the questions raised here have not been addressed.

3.3 Laser Doppler anemometry techniques

Laser Doppler anemometer (LDA) techniques evolved rapidly during the late 1960's and 1970's, and today form one of the most important tools for the quantitative investigation of engineering flows. While other non-intrusive optical techniques are being developed, none as yet have achieved the near off-the-shelf status of modern commercial laser Doppler anemometers. There exist numerous books and review articles on the application of LDA to turbulence measurement (e.g. Durst et al. [24], Buchhave et al. [25], and George [26]). However, as George [26] makes clear, ease of use has not necessarily resulted in better measurements; in fact, perhaps the opposite has been true. This is because proper use of the LDA to make any measurement is never particularly easy. As the studies of Capp [27], Capp and George [28], and Capp et al. [4] make clear, using the LDA to obtain data of sufficient quality for the evaluation of turbulence closure hypotheses can present major challenges. Some of these will be discussed briefly in the paragraphs below.

In principle, the laser Doppler anemometer provides a direct and linear measurement of a single component of the velocity which can be determined to about one part in 10^{14}. In practice, there are a number of factors which severely limit its use in real flows. Undoubtably the most serious of these is the necessity of having particles in the flow from which the laser light can be scattered. Since the largest particles (if they are not too large) scatter the most light, they will dominate the Doppler signal unless efforts are made to discriminate against them. This is particularly unfortunate since it is also the largest particles which least follow the fluctuations of the flow; and it is this latter problem which most often limits the application of the LDA for turbulence measurement.

While the fluid mechanics of suspended particles can be quite complicated, the primary determinate of the particle's motion relative to the fluid is the balance between the inertia of the particle and the viscous forces acting on it. If the particle has a greater density than the surrounding fluid (as is usually the case), the particle‘s motion lags that of the surrounding fluid with the result that the most rapid fluctuating motions of the fluid are *not* reflected in the particle's motion. Under most conditions, the response of the particle to the changing flow environment can be characterized by a single time constant depending on the size of the particle, its density relative to the surrounding fluid, and the viscosity of the fluid (v. Buchhave et al. [25] and George [26]). Since the mass of the particle (and thus its inertia) increases as the cube of the diameter while the surface area (and hence the viscous forces on it) increases only as the square, it is easy to see why the larger particles present the greatest problems since they are least able to follow the flow.

In effect, the particle acts like a low-pass filter removing the high frequency fluctuations of the flow before they can be sensed by the LDA. In liquids, this problem is much less severe than in air because the density of the particles is much closer to that of liquids than gases. The problem of particle lag is so severe in air that there are really very few experiments where the effects can be neglected, and these are usually at very low speeds. In the jet experiments from which the data shown above were taken, great care was taken to design an experiment for which the particles could be shown to track the flow for *all* frequencies contributing to the energetics of the turbulence. This generally requires that the particle time constant be less than the *Lagrangian* Komogorov microtime given by $(\nu/\epsilon)^{1/2}$. Larger values of the particle time constant relative to the Kolmogorov microtime can be used, but the burden is on the experimenter to show that significant information has not been removed from the moments of interest. In view of this, it is really astounding how many LDA results are presented where the particle lag problem is never mentioned, much less how it was overcome or limits the results.

That the problem described above is of primary importance is obvious, since if the fluctuating flow velocity information is not available for measurement, the most sophisticated data acquisition and processing techniques will not remedy the situation. The particle lag problem is, however, not the only factor limiting the application of the LDA to the kind of turbulence measurements being discussed here. There are several problems which are intrinsic to the kind of processing applied; others which can be introduced by the instrumentation itself. These have been extensively reviewed in the references cited above, and only a few will be mentioned here. Laser Doppler signal processors fall into two categories: Continuous and Burst-mode. Continuous mode processing is usually implemented by some type of frequency tracker, and as the name implies requires sufficient seed particles to provide a nearly continuous Doppler signal. Burst-mode processing operates (ideally, at least) on the Doppler burst generated by a single particle passing through the measuring volume, and thus is at its best when the seeding density is very low. The burst-mode processor is usually either a frequency counter or Fourier signal analyzer. There have been (and continue to be) efforts to operate trackers on individual bursts and counters as continuous flow devices, in spite of the difficulties in doing so.

The fundamental problem with continuous LDA processing is that it requires that there be many particles in the volume simultaneously. It is not that this is difficult to achieve (which it is in air) that is the problem, but that each of these particles carries phase information determined by when it entered the scattering volume. The phase of the overall signal from all the particles is determined by the aggregate phase of all the particles in the volume at a given instant in time. When one particle leaves the volume, the phase of the aggregate signal changes. These phase changes look exactly like velocity fluctuations, and generally can not be distinguished from them. The problem is usually referred to as the Doppler ambiguity problem, and dominates the unsteady signal from the processor at high frequencies and low turbulence levels (v. George and Lumley [29], Adrian [30] and Buchhave et al. [25]). It should be noted that it matters not whether tracker or counter is used for processing the continuous mode signal, the Doppler ambiguity can not be separated from the velocity signal if more than one particle is in the beam at a time. For the measurement of moments and spectra, the effect of the phase fluctuations from the particles (and other causes) is to add noise to the data, thereby increasing the intensity measurements. Since the phase fluctuations can be of very wide bandwidth, the measurements can be substantially in error, especially in flows of low to modest turbulence intensity.

A second problem with continuous mode processing of LDA signals is the problem of the dropout which occurs when the Doppler signal is too weak to be detected. While this is often due to a scarcity of scattering particles, it will always arises from the phase fluctuations which cause the amplitude of the Doppler signal to go to zero intermittently. Thus dropout is also intrinsic to continuous processing, can not necessarily be eliminated by better signal quality, and must therefore be accounted for. The best approach is usually to minimize it, but an alternative approach is to conditionally sample the signal only when there is signal. Buchhave et al. [25] explore the consequences of some of the options available. Dropout can act to increase or decrease the measured moments, depending on what is causing it and how it is handled. Again it only seems reasonable to expect that experimenters at least give some indication as to how much dropout was present during their measurements.

In view of the intrinsic difficulties of continuous mode processing, burst-mode processing would seem to have some unique advantages since if only a single particle at a time is sensed, there can be no phase fluctuations. Unfortunately, the burst-mode processor has its own problems. Even when the difficult problems associated with the actual electronic determination of the Doppler frequency of a particular particle can be overcome, there remains the problem of the bias which can be introduced by assuming all realizations to be equal, and therefore to be equally weighted in statistical computation. In fact, since the particles are brought to the measuring volume by the very flow which is to be measured, the statistics of the *sampling* and *sampled* processes are not statistically independent (v. George [26]). This can lead to substantial errors in the measured moments, and especially the mean velocity. These errors vanish at very low turbulence intensities, but can dominate once the turbulence intensities are above 30 – 50 %. There have been various means proposed to correct for this so-called velocity bias, but probably the best method is simply to avoid it by using residence time weighted statistics as was done in the jet data shown above. In view of the large errors which can occur when the bias problem is ignored, there is simply no excuse for it being neglected by the experimenter, either in the experiment or the write-up.

The velocity bias discussed above is not the only source of biased statistics in burst-mode LDA applications. Others can arise from the presence of mean velocity gradients in the flow, inadequate frequency shift, and various other idiosyncrasies of the instruments used to process the signals. Some of these have been discussed by George [26], others by Edwards et al. [31]. The last word has not been spoken on the matter of bias correction, so the burst-mode LDA experimenter has both considerable risk and responsibility to honestly report what he did.

While the focus here has been on the *intrinsic* problems of the LDA, most of the same considerations mentioned above for the thermal techniques such as frequency response, and especially spatial resolution apply equally to the LDA. While the LDA can be used (with great care) in high intensity turbulence environments as a stationary measuring system, it still cannot overcome the difficulties in applying Taylor's frozen field hypothesis mentioned above. Thus, while the LDA may offer unique advantages for the measurement of moments in difficult environments, it will almost never be able to measure the dissipation directly, especially when all of the problems mentioned above are considered.

4 Summary and Conclusions

Some of the challenges presented to the experimenter by the need to verify directly the various closure hypotheses of turbulence have been reviewed. The problems were illustrated by a number of examples where carefully made experimental measurements were used to test some of the assumptions of $k - \epsilon$ and Reynolds stress models. The most serious problems were attributable to the difficulties in obtaining reliable dissipation data by direct measurement. These difficulties in turn were due to the very small scales determining the dissipation and the difficulties in making direct measurements of the velocity gradients necessary to estimate it without assuming isotropy. Other difficulties could be attributed to the inability to accurately determine the curve fits (and therefore their derivatives) in various critical regions of the flow. For the LDA moment data under investigation, this was primarily due to statistical error which could have been reduced by substantially longer record lengths.

The difficulties presented to the theoretician by experimental data which does not accurately reflect the actual flow has been illustrated by the stationary hot-wire measurements for the jet. The substantial differences from the LDA data could largely be attributed to the effect of the relatively high *local* turbulence intensities on the hot-wire. (The correctness of the LDA data was inferred from the fact that it alone satisfied the governing equations and boundary conditions assumed to govern the flow, and it was independently confirmed by flying hot-wire measurements for which the local turbulence intensities were low.) Efforts to evaluate closure hypotheses from this stationary hot-wire data, however carefully it might have been taken, would have not only been frustrating, but the conclusions misleading. Clearly, great care must be taken to avoid using measurement techniques which in any way bias the data.

Finally, the most serious sources of error for thermal and laser Doppler anemometry techniques were reviewed. Receiving particular attention were those sources of bias which were dependent on the turbulence intensity since these could most adversely affect the closure hypothesis evaluation. Other problems in measurement arising from limited spatial and temporal resolution and signal processing were also discussed briefly.

In conclusion, it should be clear that the problems presented to the experimenter by the turbulence modeller are indeed formidable. Not the least of the challenges facing him is to maintain the support for a measurement program for a long enough period to enable him to learn from his mistakes and use this knowledge to finally get it right. As the measurements above illustrate, the sources of error are not always obvious from the outset, and their elimination requires a careful interplay of experiment and theory. The incentive for the effort required is that only when such an effort is made can there ever be confidence about when turbulence models will work and when they will not.

Nomenclature

C_2 Coefficient in equation 3, dimensionless

C_2' Coefficient in equation 3, dimensionless

C_3 Coefficient in equation 3, dimensionless

C_4 Coefficient in equation 4, dimensionless

D_{ij} Defined in text following equation 3, m^2/s^3

P_{ij} Defined in text following equation 3, m^2/s^3

P $= P_{ii}/2, m^2/s^3$

k Kinetic energy of turbulence, m^2/s^2

$\vec{k}$ Wavenumber vector of disturbance, m^{-1}

U_i Mean velocity vector, m/s

u_i Fluctuating velocity vector, m/s

u Fluctuating velocity component in x-direction, m/s

u' Root mean square value of u

v Fluctuating velocity component in y-direction, m/s

w Fluctuating velocity component in z-direction, m/s

x Streamwise coordinate, m

y Cross-stream coordinate, m

z Other cross-stream coordinate, m

ϵ Rate of dissipation of turbulence energy, m^2/s^3

ν Kinematic viscosity, m^2/s

ν_t Eddy viscosity, m^2/s

ω Radial frequency, rad/s

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