An Alternative Cosmological Model for an Expanding Universe

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"*I believe that the times and distances which are to be used in the Einstein's general relativity are not the same as the times and distances which were to be provided by atomic clocks. There are good theoretical reasons for believing that that is so, and for the reason that the gravitational forces are getting weaker compared to electric forces as the world gets older.*" (Paul Dirac, Göttingen Interview, 1982 [1])

Abstract

A model of the universe is postulated in which space and time expand together. It is Minkowski in one coordinate system, say $\tau, \vec{\eta}$; but our usual physical coordinates, say $t, \vec{x}$, must be scaled by a time-dependent length scale, $\delta(t)$. The Ricci tensor and Ricci scalar both vanish identically in both spaces so there is no curvature. As a result the Einstein Field equations reduce to a balance between the time-dependent spatially averaged stress energy tensor, $T_{\mu\nu}$ and its scalar invariant, $T/2$, times the metric tensor. T is shown in turn to be uniquely determined by the gravitational constant, G, the speed of light, c, and $\delta(t)$. The divergence of $T^{\mu\nu}$ is not zero, so energy is not conserved. The divergence of $T^{\mu\nu} - (T/2)g^{\mu\nu}$ is zero, however, and the conserved quantity is $G_* = \rho c^2/G\delta^2(t)$ where ρ is the rest mass

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density and G_* is Quantum Field Theory prediction – the so-called '*Worst Prediction in the History of Physics*'.

The implications of this single time-dependent length scale hypothesis for our physical space, say $(t, \vec{x})$, are explored using the rules of tensor analysis. These imply that the length scale grows exponentially with τ and linearly with t. Since the universe is assumed infinite, this is just the length scale and the visible horizon which is different for each observer. The Hubble parameter is just $H(t) = 1/t$ where t is the age of the universe. Thus the universe expansion rate is slowing down, not speeding up. The Hubble paramenter can be expressed in terms of the red-shift parameter, z, as $\tilde{H}(z(t)) = H_o\ [1 + z]$ where H_o is its current value. A value of $H_o = 63.6$ km/s/Mpc is shown to provide excellent agreement with a large number of observations. This implies that the universe began 15.4 billion years ago.

An important conclusion is that there is no need for dark energy to feed the expansion, since **neither mass nor energy are conserved quantities.** Excellent agreement is demonstrated with recent astronomical measurements, including supernovae, clusters, and the cosmic background data. These are the same data previously used to argue for Dark Energy and an accelerating universe. Finally a roadmap for astronomers is presented for analysis (or re-analysis) of existing and future data.

Contents

Nomenclature

$\Gamma^{\alpha}_{\mu\nu}$	Christoffel symbol of second kind
Δ	text used to indicate difference between two quantities; e.g. equation 71 or (75)
$\delta(t)$	length scale as function of t
$\dot{\delta}$	$= d\delta/dt$
$\ddot{\delta}$	$= d^2\delta/dt^2$
$\tilde{\delta}(\tau)$	length scale as function of $\tau; = \delta(t(\tau))$
ϵ	tiny number to account for fraction of time quantum mechanics was important relative to the age of the universe, $\epsilon = c\ t_P/R_o$
η^μ	contravariant coordinate in $\tau, \vec{\eta}$-space (defined by equation 9; $= x^\mu/\delta$
η_μ	covariant coordinate in $\tau, \vec{\eta}$-space; $= x_\mu/\delta$
η_r	radial coordinate in $\tau, \vec{\eta}$-space; $= \sqrt{\eta^i \eta_i}$
$\vec{\eta}_P$	position of particle along a path in $\tau, \vec{\eta}$-space
ϕ_e	represetative wavefunction in equation 58
κ	wavenumber
κ_{peak}	$2\pi/\lambda_m$
Λ	Einstein's cosmological constant
λ	wavelength of light
λ_a	affine parameter along path
λ_m	wavelength of maximum of blackbody spectrum
$\rho(t)$	mass density (averaged over many galaxies) as function of t
$\tilde{\rho}(\tau)$	local mass density (averaged over many galaxies) as function of τ
ρ_c	critical density in FLRW theory, defined by equation 3
ρ_o	mass density of universe at present time
σ	Stefan-Boltzmann constant
τ	dimensionaless time defined by equation 8; also proper time (equation 12)
$\tilde{\tau}_{0'0'}$	'proper time' based only on $g_{0'0'}$ (equation 15)
Ω	density ratio in FLRW theory, defined by equation 2
Ω_m	density ratio of density of baryonic matter (only) to critical density in FLRW theory
Ω	parameter in FLRW theory defined by equation 4
∇, ∇_μ	gradient operator; covariant derivative

a	2.82, needed to convert blackbody spectrum to dimensionless form using k_{peak} in equation 59
$\vec{B}$	magnetic field vector
c	speed of light
b	Wien's displacement constant
D	distance from earth to a star or galaxy or cluster
$\vec{E}$	electric field vector
$F^{\mu\nu}$	Field tensor for Maxwell's equations in curvilinear form; equation 42
F_T	blackbody spectral constant defined by equation 61
G	universal gravitational constant
G_*	universal dimensionless constant, $= \rho G \delta(t)^2/c^2 = \rho_o t_o^2 G$
$g^{\mu\nu}$	contravariant metric tensor in $\tau, \vec{\eta}$-space
$g^{\mu'\nu'}$	contravariant metric tensor in $t, \vec{x}$-space
$g_{\mu\nu}$	covariant metric tensor in $\tau, \vec{\eta}$-space
$g_{\mu'\nu'}$	covariant metric tensor $t, \vec{x}$-space
H	Hubble parameter, equation 66
$H(t)$	Hubble parameter as a function of t
$\tilde{H}(z)$	Hubble parameter as a function of redshift parameter, z; equation 76
$\mathcal{J}$	current density; equation 44
m	relative magnitude of star, equation 85
M	absolute magnitude of star
M_o	mass of visible universe at present time t_o equation 121
$\tilde{M}_g(r)$	Mass of visible universe looking back in time as a function of radius; equation 124
$M_g(z)$	Mass of visible universe looking back in time as a function of z; $M_g(z) = \tilde{M}_g(r(z))$
$\tilde{N}(r)$	cumulative number of clusters as a function of r.
$N(z)$	cumulative number of clusters as a function of z.
$\tilde{n}(r)$	number of clusters per unit volume as function of r.
$n(t)$	number of clusters per unit volume at at any time t.
p	pressure
q	charge density
r	radial coordinate: $= \sqrt{x^2 + y^2 + z^2}$
R_o	radius of visible universe at present time, t_o; $= c\ t_o$

$R^{\mu\nu}$	contravariant Ricci tensor in $\tau, \vec{\eta}$-space
$R^{\mu'\nu'}$	contravariant Ricci tensor in $t, \vec{x}$-space
$R_{\mu\nu}$	covariant Ricci tensor in $\tau, \vec{\eta}$-space
$R_{\mu'\nu'}$	covariant Ricci tensor in $t, \vec{x}$-space
$R_{\mu\nu\alpha\beta}$	covariant Riemann tensor in $\tau, \vec{\eta}$-space
$s(\vec{\eta}, \tau)$	time-varying source in equation 57
t	time in stretched $t, \vec{x}$-space (physical space); equation 24
t_r	time when radiation and total mass densities are equal; equation 112
t_1	virtual origin of theory, lower limit of equation 8 relating τ and t, time of Big Bang
t_P	Planck time scale
t_{QFT}	time theory equals QFT result
T	Invariant of stress energy tensor defined by equation 6
$T^{\mu\nu}$	contravariant stress-energy tensor in $\tau, \vec{\eta}$-space
$T^{\mu'\nu'}$	contravariant stress-energy tensor in $t, \vec{x}$-space
u_r	radial velocity in $t, \vec{x}$-space; equation 16
V_o	volume of visible universe at present time; $= 4\pi R_o^3/3$
$V(z)$	volume looking back in time as function of z; equation 123
$\vec{V}_P$	$= d\vec{\eta}_P/dt$
$\vec{x}_P$	position of particle along path in $t, \vec{x}$-space
v_r	recessional speed in Hubble relation; equation 65
x^μ	contravariant coordinate in $\tau, \vec{\eta}$-space
x^μ	contravariant coordinate in $\tau, \vec{\eta}$-space
x	$= x^{1'}$
x^μ	contravariant coordinate in $\tau, \vec{\eta}$-space
x^μ	contravariant coordinate in $\tau, \vec{\eta}$-space
x^μ	contravariant coordinate in $\tau, \vec{\eta}$-space
$x^{\mu'}$	contravariant coordinate in $t, \vec{x}$-space
y	$= x^{2'}$
Z	$= \delta^8;$ determinant of contravariant metric tensor
$\sqrt{Z}$	4-dimensional volume element
z	$= x^{3'}$

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1 Introduction

Einstein's Field Equations were first presented over the period from 1914-1917. These have been collected in Volume 6 of his collected papers [7]. Originally Einstein (and most of the astronomical community) believed the universe to be static, but he could only find static solutions to the universe by introducing a cosmological constant. Shortly after hearing of Einstein's work, both Friedman [8] and LeMaitre [9] proposed unsteady solutions which suggested the universe might be expanding. Subsequent astronomical observations by Hubble and co-workers [10] confirmed that indeed it was. In a paper with De Sitter [11], Einstein finally recognized the essential correctness of the unsteady solutions. Einstein's failure to recognize early on the implications of his equations was regarded by him as his greatest mistake. This history and theory has been discussed in thousands of places, among the most useful in this context [12, 13]. The recent historical review [14] of the Einstein/De Sitter collaboration is particularly relevant to this paper.

The prevailing model for the universe is based on the so-called Friedman-LeMaitre-Robertson-Walker equations. It is these equations together with the astronomical observations claiming that the universe is expanding at an accelerating rate that have been used to argue for the need for dark energy. They have been discussed in detail in many places as well; here we present equations 8.74 to 8.76 of Carroll [13] which nicely summarize the critical density problem. He defines a *density parameter*, Ω and a *critical density*, ρ_c as:

$$\Omega = \frac{8\pi G}{3H^2}\rho = \frac{\rho}{\rho_c} \tag{2}$$

$$\rho_c = \frac{3H^2}{8\pi G} \tag{3}$$

where ρ is the mass (or energy) density of matter in space, G is the gravitational constant, H is the Hubble parameter. These together with the FLRW equations imply:

$$\Omega - 1 = \frac{\kappa_U}{H^2 a^2} \tag{4}$$

where a is the expansion rate parameter in the FLRW-model and κ_U is the curvature of space.

The problem is that all of the available astronomical evidence (since the Hubble telescope especially) suggests that space is nearly flat, so $\kappa_U = 0$.

But all of the observed matter suggests that $\Omega = \rho/\rho_c \approx 1/3$. Clearly both cannot be true. So either the model is wrong. Or there must be some as yet unobserved matter (or energy) to make up the difference, or both. Hence the search for *dark energy* and *dark matter*. (Note that mass and energy can be considered equivalent since for rest mass, $e = \rho c^2$.) This is the cosmological crisis of our generation.

In this paper we consider an alternative to the prevailing theory (v. chapter 8 of [13] for a comprehensive presentation). We propose that space is indeed flat and infinite and expanding, but needs neither dark energy nor dark matter to explain it. Moreover for our theory the universe is in fact decelerating. Yet we argue that it agrees with the preponderance of the same astronomical observations previously used to conclude that it is accelerating [15, 5]. And it agrees with recent Hubble, Cosmic Background and galaxy number observations as well [4, 16, 3, 2]. Most importantly, our theory provides a direct link to the Quantum Field Theory (QFT) estimates (the so-called "Worst predictions in the history of physics"); and in fact treats it as the initial condition.

Finally, in this first version of our work we have deliberately put in as much detail as possible with the goal of making it straightforward to understand, even if tedious to read. Given the importance of the conclusions and our newness to the field (see Postscript), we've tried to make our analysis as transparent as possible so it should be easy to identify where we might have gone wrong. So hopefully any graduate student should be able to follow our analysis. But it should be possible as well, for those who find math boring, to simply skip over the math and go to the principle conclusions and deductions beginning with Section 6. Or for those who don't like to read at all, simply skip to the figures. They speak for themselves.

2 An alternative approach to the Einstein field equations

Instead of seeking a solution in which space alone expands, a solution of the Einstein equations is sought for the universe in which both time and space expand together.

2.1 A complete similarity solution

In the tradition of fluid mechanics and turbulence (c. f. [17, 18]), a similarity solution is sought of Einstein's Field Equations in the form [12, 13]:

$$R^{\mu\nu} = \frac{8\pi G}{c^4}\left[T^{\mu\nu} - \frac{1}{2}T\ g^{\mu\nu}\right] \tag{5}$$

where $R^{\mu\nu}$ is the Ricci tensor, $T^{\mu\nu}$ is Einstein's stress energy tensor, $g^{\mu\nu}$ is the metric tensor, c is the speed of light, and T is the scalar invariant defined by:

$$T = g_{\mu\nu}\ T^{\mu\nu} = g^{\mu\nu}\ T_{\mu\nu}. \tag{6}$$

The indices μ and ν can take values $0, 1, 2$and 3. Note for future use that as written here the contravariant stress energy tensor, $T^{\mu\nu}$, has dimensions of density times velocity squared. We also note for future consideration that we do not require the divergence of the stress energy tensor be zero, so T is not related to the Ricci scalar, $R = g^{\mu\nu}R^{\mu\nu}$, as used in [13]. Hence our use of equation 5 is not restricted to empty space if the Ricci scalar turns out to be zero.

We hypothesize the existence of a spatially homogeneous universe which in similarity coordinates, say $(\tau, \vec{\eta})$, is completely at rest; meaning that all averaged velocities (over many galaxies) in these coordinates are zero. We require that the metric describing this $\tau, \vec{\eta}$-space is Minkowski; i.e.,

$$d\tilde{s}^2 = -d\tau^2 + d\eta_\mu d\eta^\mu = -d\tau^2 + g_{\mu\nu} d\eta^\mu d\eta^\nu \tag{7}$$

where $g_{\mu\nu} = [-1, 1, 1, 1]$. As will be clear from their definitions below, all of these variables (including $d\tilde{s}$) are dimensionless. The dimensionless time, τ, shall be assumed to be the same everywhere in the universe at an instant, and monotonically increasing in equal increments from $\tau = -\infty$ to the present.

2.2 Relation to physical space

We further propose that τ and $\vec{\eta}$ are related to our physical space, say $(t, \vec{x})$ by the following:

$$\tau = c\int_{t_1}^{t} \frac{dt'}{\delta(t')} \tag{8}$$

$$\vec{\eta} = \frac{\vec{x}}{\delta(t)} \tag{9}$$

where $\delta(t)$ is an unspecified length scale to be determined and t_1 is an reference time. We will choose $\tau = 0$ to correspond to the virtual origin of our theory; i.e., the time at which the Big Bang appears to have occurred looking backwards in $\tau, \vec{\eta}$-space. Clearly t_1 must be greater than zero. We shall see later (v. Section 8.4) that a convenient choice for t_1 is proportional to the Planck time, t_P, since it measures the increasing importance of gravity relative to quantum forces. The spatial coordinate, $\vec{\eta} = \vec{x}/\delta(t)$ is a true co-moving coordinate.[1]

Since we have assumed τ and $\vec{\eta}$ to be measured in equal increments, clearly these equal increments do not correspond to equal increments in t and $\vec{x}$. We note the comment of Dirac at the beginning who seems to have been thinking along the lines we have postulated. Our t should be thought of as a gravitational or mechanical time which governs our physical laws. The dimensionless time τ, on the other hand, with its equal increments very much resembles an atomic clock. But τ could be measured in any convenient way; for example by using the temperature of the universe presumed to be cooling (suggestion of [19]). We shall do exactly this is Section 6.5 below.

2.3 The metric tensor

Note that throughout this paper we use the symbols $c\ t, \vec{x}$ and $x^{\mu'}$ interchangeably to simplify the discussions below. And similarly, $\tau, \vec{\eta}$ is used interchangeably with x^{μ}. Our use of primed and unprimed indices follow the convention of [20]. Repeated Greek indices are assumed summed over 0, 1, 2 and 3 with 0 representing time; and repeated Latin indices are summed only over 1,2 and 3. The mathematical details along with the basis vectors and Jacobians in physical space and the Christoffel symbols are included in the appendix. Here we list only the metric tensors in physical space.

The covariant metric tensor, say $g_{\mu'\nu'}$ can be computed to be:

[1]Note there is some ambiguity about this in many discussions of the FLRW metric, where the spatial variable, $\vec{x}$ is sometimes claimed to be a co-moving coordinate, but treated mathematically as if it were not. If $\vec{x}$ is a true independent variable, then the relation to our variables is $x^i/\delta(t) = a(t)x^i$. But if $\vec{x}$ is interpreted in FLRW-space [13] as co-moving it must be (and is not usually) differentiated with respect to time and space. Our definition of η^i as co-moving independent coordinate avoids this ambiguity.

$$g_{\mu'\nu'} = \frac{1}{\delta^2}\left\{\begin{array}{cccc} -1 + (\dot\delta/c\delta)^2[x^2+y^2+z^2] & -(\dot\delta/c\delta)x & -(\dot\delta/c\delta)y & -(\dot\delta/c\delta)z \\ -(\dot\delta/c\delta)x & 1 & 0 & 0 \\ -(\dot\delta/c\delta)y & 0 & 1 & 0 \\ -(\dot\delta/c\delta)z & 0 & 0 & 1 \end{array}\right\}, \quad (10)$$

where $\dot\delta = d\delta/dt$. The determinant is $g = -1/\delta^8$.

The corresponding contravariant metric tensor in physical space, $g^{\mu'\nu'}$, is readily computed to be:

$$g^{\mu'\nu'} = \delta^2\left\{\begin{array}{cccc} -1 & -(\dot\delta/c\delta)x & -(\dot\delta/c\delta)y & -(\dot\delta/c\delta)z \\ -(\dot\delta/c\delta)x & 1 - (\dot\delta/c\delta)^2x^2 & -(\dot\delta/c\delta)^2xy & -(\dot\delta/c\delta)^2xz \\ -(\dot\delta/c\delta)y & -(\dot\delta/c\delta)^2xy & 1 - (\dot\delta/c\delta)^2y^2 & -(\dot\delta/c\delta)^2yz \\ -(\dot\delta/c\delta)z & -(\dot\delta/c\delta)^2xz & -(\dot\delta/c\delta)^2yz & 1 - (\dot\delta/c\delta)^2z^2 \end{array}\right\} \quad (11)$$

It's determinant is $1/g = -\delta^8$.

2.4 Proper time and its implications

We note that the definition of proper time is given by the following integral [13]:

$$\tau_{proper} = \int_{path} d\tilde{s} = \int_{path} \sqrt{-g_{\mu'\nu'}dx^{\mu'}dx^{\nu'}} \quad (12)$$

Substituting the covariant metric from equation 10 shows that τ_{proper} is exactly equal to the τ defined by equation 8. This is not surprising since we obtained the metric from our definitions of τ and $\vec{\eta}$ in physical variables.

Interestingly, if we use only the $g_{0'0'}$ contribution to compute a 'proper time', say $\tilde{\tau}_{0'0'}$, we obtain a result that looks more like special relativity; i.e.,

$$\tilde{\tau}_{0'0'} = \int_{path} \sqrt{-g_{0'0'}}\, dx^{0'} \quad (13)$$

$$= c\int_{t_1}^{t}\left[1 - \left(\frac{\dot\delta}{c}\frac{r}{\delta}\right)^2\right]^{1/2}\frac{d\bar{t}}{\delta} \quad (14)$$

$$= c\int_{t_1}^{t}\left[1 - \left(\frac{u_r}{c}\right)^2\right]^{1/2}\frac{d\bar{t}}{\delta} \quad (15)$$

where we have used a result we shall derive later that the mean radial expansion velocity in physical space is:

$$u_r = \frac{\dot{\delta}\ r}{\delta} \tag{16}$$

In subsequent analysis we will use only the proper time defined using the entire metric tensor. This is of course our original definition of equation 8.

We can see then that the basic premise of this paper is that we think we are living in one space ($\tau, \vec{\eta}$- space). But we are really living in the other ($t, \vec{x}$-space). We shall explore this difference in detail. And we shall show that at least some of what believed to be true probably was not. But the data were correct; only the interpretations of at least some of it was not.

2.5 The Riemann and Ricci tensors and Ricci scalar

It is straightforward (e.g.,using Maple or Mathematica) to show that the Riemann and Ricci tensors and the Ricci scalar are identically zero. This is no surprise, since the original $\tau, \vec{\eta}$-space was assumed to be flat (Minkowski). So the transformed space must be as well.

Clearly an expanding universe does not require a Riemann tensor with curvature. In our quasi-equilibrium model, the universe has no average motion in its own $(\tau, \vec{\eta})$-coordinates, but it does in ours $(t, \vec{x})$. Note, however, that the form of our metric tensor in physical coordinates does appear to have curvature. But these extra terms, $[(\dot{\delta}/c)|\vec{x}|/\delta]$, vanish for small distances relative $R_o = c\ t$. Even when these terms are not small, however, the Riemann tensor is identically zero.

In the following sections we shall explore first the kinematics of our assumed universe and show it is consistent with many astronomical observations. Then we shall show what the vanishing Riemann tensor implies about the stress energy tensor and explore its consequences for the dynamics. Finally we shall try to outline how our theory might be used to interpret (or reinterpret) recent observations on the distribution of mass and clusters throughout the universe.

3 The velocity

How the velocities in the two coordinate systems behave and relate to each other is clearly of prime interest. In this section we consider their behavior in both spaces. And we discover that the length scale is determined by them.

3.1 Velocity in $\tau, \vec{\eta}$-space

We first define a displacement field in our expanding-space-time coordinate system to be $\vec{\eta}_p(\tau, \vec{\eta})$. Note that we use the subscript p to distinguish the proper-time-dependent displacement field from the independent variables $\tau, \vec{\eta}$. Then the four-dimensional 'velocity' is $\vec{V}_p(\tau, \vec{\eta}) = \partial\vec{\eta}_p/\partial\tau$. But only time is passing; otherwise nothing is moving (by assumption). So the velocity in this space is simply $1, 0, 0, 0$, since only $\partial\tau_p(\tau, \vec{\eta})/\partial\tau$ is non-zero and equal to one.

3.2 Velocity in $t, \vec{x}$-space

Now what we would really like to know is how $\vec{V}_p$ looks in physical space (or $t, \vec{x}$-space). Unfortunately we cannot simply transform it like an ordinary vector, since derivatives of vectors in general do not transform like tensors. We must use Lie derivatives and their associated Christoffel symbols. Note that we did not need these for $\vec{V}_p$ in $\tau, \vec{\eta}$-coordinates since in the hypothesized Minkowski space the Christoffel symbols are all zero.

If we define $\vec{x}_p(\vec{x}, t)$ to correspond to our function $\eta_p(\tau, \vec{x})$ we can write our covariant derivative condition as:

$$\nabla_{cov}\vec{x}_p = \left\{ \frac{\partial x_p^{\nu'}}{\partial x^{\mu'}} + \Gamma^{\nu'}_{\mu'\alpha'} x_p^{\alpha'} \right\} \vec{b}_{\nu'} = 0 \tag{17}$$

where we have used primes to distinguish which set of coordinates we are using. Note that since the basis vectors are themselves time-dependent, a necessary condition for solution is that the terms in curly brackets be zero for each value of ν'.

The problem will be considerably simplified if we transform equation 17 into derivatives with respect to an affine (equally spaced) parameter, say λ_a. Since the tangent to the curve is given by $dx_p^{\mu'}/d\lambda_a$ we can multiply equation 17 by it to obtain:

$$\frac{\partial x_p^{\nu'}}{\partial \lambda_a} + \Gamma^{\nu'}_{\mu'\alpha'} \frac{dx_p^{\mu'}}{d\lambda_a} x_p^{\alpha'} = 0 \tag{18}$$

Note that the first term is simply the directional derivative. Also note that we can choose the proper time for λ_a; so we choose $\tau = \lambda_a$.

In the following subsections we shall first consider $x^{0'} = t$. Then in the next, $x^{\nu'}$ for $\nu' = 1', 2'$, and $3'$.

3.3 Covariant derivative for $\nu' = 0$

From equation 154 in Appendix A the only non-zero Christoffel symbol for $\nu' = 0$ is $\Gamma^{0'}_{0'0'}$. So equation 18 reduces to:

$$
\begin{aligned}
0 &= \frac{\partial x_p^{0'}}{\partial \lambda_a} + \left(-\frac{\dot{\delta}}{\delta}\right)\frac{\partial x_p^{0'}}{\partial \lambda_a} x_p^{0'} \\
&= \frac{\partial x_p^{0'}}{\partial \lambda_a}\left[1 - \left(\frac{\dot{\delta}}{\delta}\right) x_p^{0'}\right] \qquad (19)
\end{aligned}
$$

It is pretty obvious that the only solution is for:

$$
\frac{\dot{\delta}\ x_p^{0'}}{\delta} = 1 = \frac{\dot{\delta}\ t}{\delta} \qquad (20)
$$

since $x_p^{0'} = t$. We didn't need to use the fact that $\lambda_a = \tau$, but we will use it later.

Equation 20 can be integrated directly to yield:

$$
\frac{\delta(t)}{\delta(t_1)} = \frac{t}{t_1} \qquad (21)
$$

where $\delta(t_1)$ is the value of $\delta(t)$ at time t_1.

On dimensional grounds alone we recognize that the coefficient between $\delta(t)$ and t must be the speed of light, c. We can without loss of generality define:

$$
\delta(t) = c\ t. \qquad (22)
$$

This choice of coefficient insures that $\delta(t_P)$ is the Planck length scale if t_P is the Planck time scale. This will prove to be important later. It is humbling (but quite reassuring) to note that we could have deduced equation 22 by dimensional analysis alone.

We shall see in section 6 below that $H = \dot{\delta}/\delta$ is the Hubble parameter. So clearly $H = 1/t$. So if t_o is the age of the universe, then $1/H_o = 1/H(t_o) = t_o$ is the age as well. This is consistent with wide speculation for many decades, but not previous theories. As noted by Carroll and Ostlie [21], almost all previous "ages" of the universe have been calculated using a Friedmann model. So our "ages" and "times" will be different from those in common use.

3.4 Relation of τ and $\tilde{\delta}(\tau)$ to t and $\delta(t)$

From equations 8 and 22, the relation between our proper time τ and t reduces to just:

$$\tau = c \int_{t_1}^{t} \frac{dt'}{\delta(t')} = \ln t/t_1 \tag{23}$$

So time in our reference space is logarithmically related to our physical space time t. Note that by hypothesis, it is the increments of τ that are equally spaced, and increments of physical space time which are stretched.[2]

Alternatively, we can express equation 23 this way:

$$\frac{t}{t_1} = \exp(\tau), \tag{24}$$

Equations 22 and 23 imply that in $\tau, \vec{\eta}$-space the length scale is varying exponentially with τ; i.e.,

$$\tilde{\delta}(\tau) = \; c \; t_1 \; \exp(\tau) \tag{25}$$

where we have defined $\tilde{\delta}(\tau) = \delta(t(\tau))$ to distinguish its different independent variable.

So in t, x-space the length scale varies linearly and time logarithmically. But in $\tau, \vec{\eta}$-space, time varies linearly and the length scale grows exponentially.

[2]This suggests strongly that it is τ which is measured by atomic clocks which presumably would be unaffected by gravity. So any inference that these atomic clocks speed up or slow down to account for observations might in fact be backwards. It is the physical time which is changing, not the atomic clock.

3.5 Covariant derivative for $\nu' = 1', 2'$, and $3'$

The equations for $\nu' = 1', 2'$, and $3'$ are considerably more complicated since there are three non-zero Christoffel symbols for each. We will consider $\nu' = 1'$ first, but the others will be similar. Our task is somewhat simplified by the $\nu' = 0'$ result; we need only show here that the $\delta(t)$ we deduced in equation 20 is a solution to these equations as well.

Substituting into equation 18 using the values for $\nu' = 1'$ yields:

$$\begin{aligned} 0 &= \frac{\partial x_p^{1'}}{\partial \lambda_a} + \Gamma^{1'}_{0'0'} \frac{\partial x_p^{0'}}{\partial \lambda_a} x_p^{0'} + \Gamma^{1'}_{0'1'} \frac{\partial x_p^{0'}}{\partial \lambda_a} x_p^{1'} + \Gamma^{1'}_{1'0'} \frac{\partial x_p^{1'}}{\partial \lambda_a} x_p^{0'} \qquad (26) \\ &= \frac{\partial x_p^{1'}}{\partial \lambda_a} - x_p^{1'} \left[\frac{\ddot{\delta}}{\delta} - \left(\frac{\dot{\delta}}{\delta} \right)^2 \right] \frac{\partial x_p^{0'}}{\partial \lambda_a} x_p^{0'} - \left(\frac{\dot{\delta}}{\delta} \right) \frac{\partial x_p^{0'}}{\partial \lambda_a} x_p^{1'} - \left(\frac{\dot{\delta}}{\delta} \right) \frac{\partial x_p^{1'}}{\partial \lambda_a} x_p^{0'} \\ &= \frac{\partial x_p^{1'}}{\partial \lambda_a} \left[1 - \left(\frac{\dot{\delta}}{\delta} \right) x_p^{0'} \right] - \frac{\partial x_p^{0'}}{\partial \lambda_a} x_p^{1'} \left\{ \left(\frac{\dot{\delta}}{\delta} \right) + \left[\frac{\ddot{\delta}}{\delta} - \left(\frac{\dot{\delta}}{\delta} \right)^2 \right] x_p^{0'} \right\} \end{aligned}$$

The first term in square brackets is clearly zero if we substitute the result of equation 20. It is easy to show by differentiation that this result also makes the curly bracketed term on the right zero as well.

So what is the $1'$-component of velocity, say $u_p^{1'}$? It is given by:

$$u_p^{1'} = \frac{\partial x_p^{1'}}{\partial x_p^{0'}} = \frac{\partial x_p^{1'}}{\partial \lambda_a} \frac{\partial \lambda_a}{\partial x_p^{0'}} = \frac{\partial x_p^{1'} / \partial \lambda_a}{\partial x_p^{0'} / \partial \lambda_a} = x_p^{1'} \left(\frac{\dot{\delta}}{\delta} \right) \left\{ \frac{1 - \dot{\delta} x_p^{0'} / \delta}{1 - \dot{\delta} x_p^{0'} / \delta} \right\} \qquad (27)$$

The last expression can be obtained by factoring out $\dot{\delta}/\delta$ and substituting using equation 20. It ends up being the ratio of two terms that are zero but exactly the same. So we are left with:

$$u_p^{1'} = x_p^{1'} \left(\frac{\dot{\delta}}{\delta} \right). \qquad (28)$$

Similar equations result for $i' = 2'$ and $3'$. So we have:

$$u_p^{i'} = x_p^{i'} \left(\frac{\dot{\delta}}{\delta} \right). \qquad (29)$$

This is the result (equation 16) we used in Section 2.4 above. Note the material derivative of equation 29 is zero, consistent with our assumed rest state in $\tau, \vec{\eta}$ variables.

So for all values of $\mu' = 0, 1, 2$ and 3, the solution to equation 17 is just:

$$u_p^{\mu'} = x_p^{\mu'} \left(\frac{\dot{\delta}}{\delta} \right). \tag{30}$$

4 Accelerations and the Geodesic Equation

In general relativity it is the geodesic equation which provides the counterpart to Newton's Law. In this section we derive it for our proposed universe.

4.1 Geodesic equation

If $x_p^{\mu}(\lambda_a)$ is a world-line (particle trajectory), then the trajectory that minimizes $\tilde{\tau}$ is given by the *geodesic equation*:

$$\frac{d^2 x^{\mu}(\lambda_a)}{d\lambda_a^2} + \Gamma^{\mu}_{\alpha\beta} \frac{dx^{\alpha}(\lambda_a)}{d\lambda_a} \frac{dx^{\beta}(\lambda_a)}{d\lambda_a} = 0 \tag{31}$$

Note that in Einstein's interpretation it is the negative of the second term on the left-hand-side (the one with the Christoffel symbol) which represents the gravitational potential gradient.

4.2 Writing the geodesic equation with proper time.

We choose the proper time, τ defined by equation 8 as our affine parameter, noting that it is the same as the proper time defined by equation 12.

So we can re-write our geodesic equation using proper time as:

$$\frac{d^2 x_p^{\mu'}(\tau)}{d\tau^2} + \Gamma^{\mu'}_{\alpha'\beta'} \frac{dx_p^{\alpha'}(\tau)}{d\tau} \frac{dx_p^{\beta'}(\tau)}{d\tau} = 0 \tag{32}$$

Note that we have also replaced $x^{\mu}(\tau)$ by $x_p^{\mu'}(\tau)$ to emphasize that the latter is NOT an independent variable, but instead a parameterized path. And we have used primes on the indices since these are in our primed coordinates. Note that $x_p(\tau)$ is a hybrid quantity. It is the parameterized spatial displacement in our physical coordinates, but written as a function of τ, not t.

So $x_p(\tau) = \delta(t)\ \eta_p(\tau)$ where $\eta_p(\tau)$ is the parameterized path in $\tau - \vec{\eta}$-space. Finally recall that $x^{0'} = t$, so:

$$\frac{dx_p^{0'}(\tau)}{d\tau} = \frac{dt_p(\tau)}{d\tau} = \frac{1}{d\tau/dt} = \frac{\delta}{c}. \tag{33}$$

We can expand equation 32 for $\mu' = 1'$ to obtain:

$$\begin{aligned} 0 \;=\; & \frac{d^2x_p^{1'}(\tau)}{d\tau^2} + \Gamma^{1'}_{0'0'}\frac{dx_p^{0'}(\tau)}{d\tau}\frac{dx_s^{0'}(\tau)}{d\tau} \qquad (34) \\ & + \Gamma^{1'}_{1'0'}\frac{dx_p^{1'}(\tau)}{d\tau}\frac{dx_p^{0'}(\tau)}{d\tau} + \Gamma^{1'}_{0'1'}\frac{dx_p^{0'}(\tau)}{d\tau}\frac{dx_p^{1'}(\tau)}{d\tau} \end{aligned}$$

where we have kept only the non-zero Christoffel symbols. Substituting for these and using equation 33 above yields:

$$0 \;=\; \frac{d^2x_p^{1'}(\tau)}{d\tau^2} + \Gamma^{1'}_{0'0'}\left[\frac{\delta}{c}\right]^2 + \left[\Gamma^{1'}_{1'0'} + \Gamma^{1'}_{0'1'}\right]\frac{\delta}{c}\frac{dx_p^{1'}(\tau)}{d\tau} \tag{35}$$

Substitution for the Christoffel symbols and re-arranging yields:

$$0 = \frac{d^2x_p^{1'}(\tau)}{d\tau^2} - 2\left[\frac{\dot{\delta}}{c}\right]\frac{dx_p^{1'}(\tau)}{d\tau} - \left[\frac{\delta\ddot{\delta} - (\dot{\delta}^2)}{c^2}\right]x_p^{1'}(\tau) \tag{36}$$

There are identical equations for $\mu' = 2'$ and $3'$. So combining all three yields:

$$0 = \frac{d^2x_p^{i'}(\tau)}{d\tau^2} - 2\left[\frac{\dot{\delta}}{c}\right]\frac{dx_p^{i'}(\tau)}{d\tau} - \left[\frac{\delta\ddot{\delta} - (\dot{\delta}^2)}{c^2}\right]x_p^{i'}(\tau) \tag{37}$$

where $i' = 1, 2$ and 3.

4.3 Showing that the geodesic equation is solved.

Our problem is that the coefficients are dependent on t, while the $\vec{x}_p$'s are functions of $\vec{\eta}$ and τ. We know that $x_p(\tau) = \delta(t)\ \eta_p(\tau) = \tilde{\delta}(\tau)\eta_p(\tau)$. Plus we know that $\vec{\eta}_p(\tau) = \vec{\eta}_p(0)$ is time independent (by hypothesis). Thus we can write:

$$0 = \vec{\eta}_p(0)\left\{\frac{d^2\tilde{\delta}(\tau)}{d\tau^2} - \ 2\left[\frac{\dot{\delta}}{c}\right]\frac{d\tilde{\delta}(\tau)}{d\tau} - \left[\frac{\delta\ddot{\delta} - (\dot{\delta}^2)}{c^2}\right]\tilde{\delta}(\tau)\right\} \tag{38}$$

But we can use the chain-rule to convert $d/d\tau$ to $(\delta/c)d/dt$ with the result:

$$0 = \vec{\eta}_p(0)\left\{\left[\frac{\delta}{c}\right]\frac{d}{dt}\left(\left[\frac{\delta}{c}\right]\frac{d}{dt}\delta\right) - \ 2\left[\frac{\dot{\delta}}{c}\right]\left[\frac{\delta}{c}\right]\frac{d}{dt}\delta - \left[\frac{\delta\ddot{\delta} - (\dot{\delta}^2)}{c^2}\right]\delta\right\} \tag{39}$$

Since this equation must be satisfied for any $\vec{\eta}_p(0)$, the term in curly brackets on the right-hand-side must be itself identically zero. The bracketed term reduces to:

$$0 = \left[\frac{\delta\ddot{\delta} + (\dot{\delta}^2)}{c^2}\right] - \ 2\left[\frac{\dot{\delta}}{c}\right]^2 - \left[\frac{\delta\ddot{\delta} - (\dot{\delta}^2)}{c^2}\right] \tag{40}$$

Interestingly, this is satisfied for ALL $\delta(t)$, no matter the particular functional form of $\delta(t)$. So the geodesic equation is satisfied.

5 Maxwell's equations and Wave Propagation

Almost everything we know about the universe is due to light propagation. And light propagation is governed by Maxwell's equations. Therefore we need to consider how they change with coordinate system in order to interpret them in either $(\vec{x}, t)$ or $(\tau, \vec{\eta})$-space. And we must also understand how this affects the spectra and intensities of the radiation received.

5.1 Maxwell's equations in curvilinear coordinates

Carroll [13] writes Maxwell's equations in curvilinear coordinates as:

$$\nabla_\mu \ F^{\mu\nu} \ = \mathcal{J}^\nu \tag{41}$$

where ∇_μ is the covariant derivative, $F^{\mu\nu}$ is defined as:

$$F^{0i} = E^i \tag{42}$$
$$F^{ij} = \epsilon^{ijk} B_k \tag{43}$$

for $i = 1, 2$ or 3 where $\vec{E}$ and $\vec{B}$ are the electric and magnetic field vectors respectively. The charge, q, and current, $\vec{\mathcal{J}}$, distributions have been combined into a four vector, $\mathcal{J}^\mu$, defined as:

$$\mathcal{J}^\mu = (q, \mathcal{J}^1, \mathcal{J}^2, \mathcal{J}^3) \tag{44}$$

Now all of the usual rules for tensor transformation apply. In particular, since we know the Jacobians and the metric tensors, we can easily move these from $\tau, \vec{\eta}$-space to our $t, \vec{x}$-space, or vice versa. Or from contravariant to covariant components. or vice versa. It is easy to show that the left-hand-side of the equation will look exactly the same in either coordinate system, a consequence of the linearity and of Lorentz invariance. But since $\mathcal{J}^\nu = J^\nu_{\nu'} \mathcal{J}^{\nu'}$ where $J^\nu_{\nu'}$ is the Jacobian, the source terms in one frame will differ by a factor of δ from the other. For example, if we specify a charge source distribution in the $\tau, \vec{\eta}$-frame by say $q(\vec{\eta}, \tau)$, then in the other frame it will appear multiplied by $1/\delta(t)$. Also a charge at rest in one representation will appear as a current in the other.

The same will be true for the electric and magnetic field vectors as well. Since the power radiated is the cross–product of the electric and magnetic field vectors, the power will be modified by $\delta(t)^2$. But this will exactly compensate in the inverse square law using $1/D^2$ instead of $1/\eta_D^2 = (\delta/D)^2$ where D is a distance from the source. This will have profound implications for how we interpret intensity data from an isolated source. In particular, it means that the spectrum in physical variables will be shifted in both wavenumber and amplitude, so the power is conserved. We will discuss this in more detail in Section 7.2 below where we consider *relative* intensities of radiation from supernova data. Whatever space you chose to work in, *both* intensities must be transformed, not just one. The difference is the mysterious $1 + z$ which has made the universe appear to accelerate when the opposite is true.

5.2 Re-scaling Maxwell's equations into $\tau, \vec{\eta}$-coordinates

It is easier to consider Maxwell's equation here using vector notation instead of their curvilinear coordinate representations. We can write the electric and magnetic field vectors $\tilde{\vec{E}}$ and $\tilde{\vec{B}}$ and current vector $\tilde{\vec{\mathcal{J}}}$ as functions of $\vec{\eta}, \tau$. So Maxwell's equations [22] with no loss of generality reduce to:

$$\nabla_\eta \cdot \tilde{\vec{E}} = \tilde{q}\, \tilde{\delta}(\tau) \tag{45}$$

$$\nabla_\eta \cdot \tilde{\vec{B}} = 0 \tag{46}$$

$$\nabla_\eta \times \tilde{\vec{B}} = -\left[\frac{\delta}{c}\right] \frac{\partial \vec{E}}{\partial t} + \tilde{\vec{\mathcal{J}}}\, \tilde{\delta}(\tau)) = -\frac{\partial \tilde{\vec{E}}}{\partial \tau} + \tilde{\vec{\mathcal{J}}}\, \tilde{\delta}(\tau) \tag{47}$$

$$\nabla_\eta \times \tilde{\vec{E}} = \left[\frac{\delta}{c}\right] \frac{\partial \vec{B}}{\partial t} = \frac{\partial \tilde{\vec{B}}}{\partial \tau} \tag{48}$$

where $\tilde{q}$ is the charge distribution in $\tau, \vec{\eta}$ - coordinates. We have used Gaussian variables which absorb the dielectric and permeability constants into the definitions of the field vectors. These transformed equations themselves look exactly like the old in the new variables since $\delta(t)$ (or $\tilde{\delta}(\tau)$) have been absorbed into τ. This is a consequence of their linearity. Only the source terms look different because of the length scale, $\tilde{\delta}(\tau)$.

Equations 47 and 45 can be cross-differentiated and combined to form wave equations for the magnetic and electric field vectors as follows:

$$\left[\nabla_\eta^2 - \frac{\partial^2}{\partial \tau^2}\right] \tilde{\vec{B}} = 4\pi \left\{ \left(\frac{\tilde{\delta}}{c}\right) \frac{d\tilde{\vec{\mathcal{J}}}}{d\tau} + \frac{\tilde{\vec{\mathcal{J}}}}{c} \frac{d\tilde{\delta}}{d\tau} \right\} \tag{49}$$

$$\left[\nabla_\eta^2 - \frac{\partial^2}{\partial \tau^2}\right] \tilde{\vec{E}} = 4\pi \left(\frac{\tilde{\delta}}{c}\right) \left\{ c\, \nabla_\eta \tilde{q} - \nabla_\eta \times \tilde{\vec{\mathcal{J}}} \right\} \tag{50}$$

Note that from equation 25 it follows that $\tilde{\delta} = d\tilde{\delta}/d\tau$, so the right-hand-sides of both equations are multiplied by $\delta = \tilde{\delta}$, exactly as should have been expected from the previous subsection. As noted above in the preceding subsection, this has profound implications for how we interpret intensity data from an isolated source. It also means that the spectrum in physical variables is shifted in both wavenumber and amplitude, so the power is conserved. We will discuss this in more detail in Section 7 below.

5.3 Wave propagation in $\tau, \vec{\eta}$-space

In $\tau, \vec{\eta}$-space, the transformed equations for $\tau, \vec{\eta}$-space imply that there is no change in the wavenumber, say, $\vec{\kappa_*}$ or frequency, say ω_*, since in this space nothing is expanding. So the phase of a wave propagating in $\tau, \vec{\eta}$-space from a source at $\vec{\eta}_s$ and time τ_s is given by:

$$\vartheta = \vec{\kappa}_* \cdot (\vec{\eta} - \vec{\eta}_s) - \omega_* (\tau - \tau_s). \tag{51}$$

Note that τ_s is a specific time and so not changing with time τ. But if the star is not moving at the average velocity of the universe (assumed to be zero in $\tau, \vec{\eta}$-coordinates), $\vec{\eta}_s$ is changing with τ. If we define $\kappa_* = |\vec{\kappa}_*|$, the dimensionless wavespeed is simply $c_* = \omega_* / \kappa_*$ and equal to unity.

What does this look like in our physical universe. We will see these waves in our physical space propagate at frequency and wavenumber given by:

$$\nu \;=\; -\frac{\partial \vartheta}{\partial t} \;=\; \frac{\omega_*}{\delta} \;-\; \vec{\kappa_*} \cdot \left(-\vec{\eta} \frac{\dot{\delta}}{\delta} + \frac{d\vec{\eta}_s}{dt} \right) \tag{52}$$

$$\vec{k} \;=\; \nabla \vartheta \;=\; \frac{\vec{\kappa}_*}{\delta(t)}. \tag{53}$$

The last term in parentheses in equation 52 will be identically zero for a particular star moving with the local average velocity of the universe. And it will be zero by hypothesis for an average of them over a large enough field, our continuum (or field) hypothesis. We will use equation 53 extensively to analyze the Hubble measurements in Section 6.2 below.

Interestingly, the shift in wavenumber is completely independent of velocity, and depends only on the length scale $\delta(t)$. This is consistent with the observations of [13] and others that the *average* redshift is not really a Doppler shift at all. But the total redshift, however, of an individual body is a consequence of both the expansion of the universe *and* the relative motion of a particular body in it.

5.4 Radiation from a point source into $\tau, \vec{\eta}$-space

Any of the wave equations above, equations 49 and 50, can be written in the form:

$$\Box_{\eta.\tau}\phi_e(\vec{\eta},\tau) = s(\vec{\eta},\tau) \tag{54}$$

where s is a source and where $\Box$ is the d'Alembertian defined as:

$$\Box_{\eta.\tau} = \nabla_\eta^2 - \frac{\partial^2}{\partial\tau^2} \tag{55}$$

We are only interested here in the radially symmetrical solution which reduces to:

$$\left[\frac{1}{\eta_r^2}\frac{\partial}{\partial\eta_r^2}\left(\eta_r^2\frac{\partial}{\partial\eta_r}\right) \;-\; \frac{\partial^2}{\partial\tau^2}\right]\phi_e \;=\; s \tag{56}$$

where we have defined the radial coordinate to be $\eta_r = |\vec{\eta}|$.

Since the stars of interest are at great distance, they can be modelled with a simple time varying source and a delta-function; i.e.,

$$s(\vec{\eta},\tau) = A_s\ \delta_D(\vec{\eta}-\vec{\eta}_s,\tau-\tau_s)\ \ exp(i\ [\vec{\kappa}_*\cdot\vec{\eta}-\omega_*\tau]) \tag{57}$$

where $\vec{\eta}_s, \tau_s$ is the location of the star at time τ_s and $\delta_D(\vec{\eta},\tau)$ is a four dimensional Dirac-delta-function. (Note this δ-function should not be confused with our length scale.) For the moment we consider only a single frequency, the frequency of the source, ω_*; and we allow the wave to propagates radially through space with wavenumber, κ_* as described in the preceding section.

By transforming equation 56 in time to obtain a Helmholtz equation, the result can be directly solved to the usual point source retarded-time solution to Maxwell's wave equations given by:

$$\phi_e(\vec{\eta},t) = \frac{1}{4\pi\eta_r}e^{i(\vec{\kappa}\cdot[\vec{\eta}-\vec{\eta}_s]-c[\tau-\tau_s])} \tag{58}$$

This means that the intensity of the radiation from a star in $\tau,\vec{\eta}$-coordinates will obey an inverse square law in $|\vec{\eta}|$, and the frequencies and wavenumbers will remain unchanged from the source. In the physical space, however, the frequency and wavenumber detected at distances away from the source will be given instead by equations 52 and 53. But, somewhat surprisingly, as noted in Sections 5.1 and 5.2, the intensities in physical space will drop off as an inverse square law in r, not $r/\delta(t)$. *So no "corrections' to intensities are necessary in either space.*

5.5 Blackbody radiation in an expanding universe

To examine how spectra change with distance we can consider how the spectrum of a blackbody is modified with distance from the emitting object. Since Maxwell's equations are linear, we can simply superimpose solutions and consider a whole spectrum of frequencies (or wavenumbers) summed together. So we define $F(k)$ where $k = |\vec{k}|$ to be the power emitted per unit projected area of the blackbody at temperature T into a solid angle in the wavenumber interval from k to $k + dk$:

$$F(\tilde{k}) = F_T \left[\frac{k}{k_{peak}}\right]^3 \left[\exp\left(a\frac{k}{k_{peak}}\right) - 1\right]^{-1} \tag{59}$$

where the constant a can be shown by integration to be 2.82.

$$k_{peak} = \frac{2\pi}{\lambda_m} = \frac{2\pi}{b}\, T_s \tag{60}$$

where T_s is the star temperature and b is Wien's displacement constant, and we have defined F_T to be:

$$F_T \;=\; 2\;h\;c^2\;k_{peak}^3 \;=\; 2h\;c^2\left[\frac{2\pi\;T_s}{b}\right]^3 \tag{61}$$

Note that $F_T \propto T_s^3$, while the spectral peak wavenumber is proportional to T_s.

So we can define a dimensionless wavenumber, say $\overline{k} = k/k_{peak}$, and write the non-dimensional similarity black-body spectrum at the time, say t, when the radiation was emitted like this:

$$F(k,t) = F(\overline{k}\;k_{peak}, t) = F_T\;\left\{\frac{\overline{k}^3}{\exp(a\overline{k}) - 1}\right\}. \tag{62}$$

The total power emitted per unit projected area into a solid angle at any time t is given by:

$$\begin{aligned} I(t) &= F_T\;\;k_{peak}\int_0^\infty \frac{\overline{k}^3}{e^{a\overline{k}} - 1}\;\;d\overline{k} \\ &= \frac{\pi^4}{15\;a^4}\;F_T\;k_{peak} \\ &= h\;c^2\;\frac{2\;\pi^4}{15\;a^4}\;\left(\frac{2\pi T_s}{b}\right)^4 \end{aligned} \tag{63}$$

This is the energy flux of the radiation at the source, and it clearly depends only on the temperature T_s.

Now we can see HOW this spectrum is propagated through $\tau, \vec{\eta}$-space by using the dimensionless wavenumber, κ, we defined in equation 53:

$$\kappa \;\; = \;\; k\ \delta(t) \; = \; \overline{k}\ k_{peak}\ \delta(t) \tag{64}$$

So the entire spectrum shifts to lower physical space wavenumbers as $\delta(t)$ increases. But it does so in a way as to preserve an inverse square law.

6 Hubble's Law

Hubble's law and the observations of it have dominated discussions in astronomy for almost a century, and never more than now [23]. In this section we show that our derived relation with the redshift parameter (defined in equation 69 below) is in excellent agreement with the recent data.

6.1 Derivation of Hubble's law

Assume D to be the distance to some distant star or galaxy so that its normalized distance in $\tau, \vec{\eta}$-space is $|\vec{\eta}_p| = D/\delta(t)$. *Since we are computing derivatives of something that happened long ago, both the distance and the time should be evaluated using values corresponding to the time when the information was transmitted, say t_s, not when it is received.* For distant galaxies t_s is much earlier than our present time, say t_o.

Also we assume any 'local' deviation velocity of the object, say $|\Delta\vec{V}_p|$ is negligible compared to the mean velocity of the expanding universe (i.e., $|\Delta V_p| << HD$), or at least 'averages out' when many neighboring objects are considered. Equation 16 implies immediately that the recessional speed from us, say v_r is given by:

$$v_r = \dot{\delta}\ \frac{D}{\delta} = H\ D \tag{65}$$

where we have defined H to be:

$$H \equiv \left[\frac{1}{\delta}\frac{d\delta}{dt}\right] = \frac{\dot{\delta}}{\delta}. \tag{66}$$

Equation 65 looks like Hubble's law. But it is exactly Hubble's Law only if H is a constant. We have deduced that $\delta(t) \propto t$, so:

$$H(t) = \frac{1}{t} \tag{67}$$

where t is the 'age of the universe' as measured from some virtual origin at the time $H(t)$ is evaluated.

6.2 The relation between H and z

The time-dependent wavelength, say $\lambda(t)$, is related to the time-dependent wavenumber by $k = 2\pi/\lambda$. The wavelength transmitted from a star at time time t_s we denote as λ_s; and we assume it to be equal to the wavelength it would have had on earth. We will denote the present time by t_o and the wavelength when it arrives on earth as λ_o. So the change in wavenumber between transmission and reception is:

$$\begin{aligned} \Delta k &= \left\{ \frac{2\pi}{\lambda_s} - \frac{2\pi}{\lambda_o} \right\} \\ &= \frac{2\pi}{\lambda_o} \left[\frac{\lambda_o - \lambda_s}{\lambda_s} \right] \end{aligned} \tag{68}$$

We define the redshift parameter, z, by:

$$z = \frac{\lambda_o - \lambda_s}{\lambda_s} = \frac{\Delta\lambda}{\lambda_s} \tag{69}$$

So $\lambda_o = \lambda_s \left[1 + z\right]$. And we can rewrite equation 68 as:

$$\Delta k = \left\{ \frac{2\pi}{\lambda_s} - \frac{2\pi}{\lambda_o} \right\} = \frac{2\pi}{\lambda_s} \left[\frac{z}{1+z} \right] \tag{70}$$

Also we have using equation 53:

$$\Delta k = \left\{ \frac{\kappa_s}{\delta(t_s)} - \frac{\kappa_s}{\delta(t_o)} \right\} = \frac{\kappa_s}{\delta(t_s)} \left\{ 1 - \frac{\delta(t_s)}{\delta(t_o)} \right\} = \frac{2\pi}{\lambda_s} \left\{ 1 - \frac{H(t_o)}{H(t_s)} \right\} \tag{71}$$

since $H(t) = 1/t$ and $\delta(t) = c\, t$ from our theory.

Equating equations 70 and 71 yields:

$$\left\{1 - \frac{H(t_o)}{H(t_s)}\right\} = \left[\frac{z}{1+z}\right] \tag{72}$$

Or:

$$\frac{H(t_o)}{H(t_s)} = 1 - \left[\frac{z}{1+z}\right] = \frac{1}{1+z} \tag{73}$$

We can solve this for $\Delta\tilde{H}(z) = H(t_s(z)) - H(t_o)$ to obtain:

$$\frac{\Delta\tilde{H}(z)}{H(t_s)} = \frac{z}{1+z} \tag{74}$$

Note that the $H(t_s)$ is evaluated at the time the light was emitted, t_s.

We can more conveniently express $\Delta\tilde{H}$ in terms of the present value $H(t_o)$:

$$\frac{\Delta\tilde{H}(z)}{H(t_o)} = z \tag{75}$$

So the actual value of the Hubble parameter for any value of z is given by:

$$\tilde{H}(z) = H(t_o)\ [1+z] = H_o\ [1+z] \tag{76}$$

where we have defined $H_o = H(t_o)$ to be consistent with conventional notation. Note that H_o is the only unknown parameter.

The simple relation above can be contrasted with the standard model result (c. f. [13, 4]):

$$H(z) = H_o\sqrt{\Omega_{mo}(1+z)^3 + \Omega_{ro}(1+z)^4 + 1 - \Omega_{mo} - \Omega_{ro}} \tag{77}$$

where Ω_{mo} and Ω_{ro} are the current values of the non-relativistic and relativistic matter density parameters.

H_o is presumed by both theories to be the present value of the Hubble parameter. It is the *only* adjustable parameter in the theory presented here, whereas there are two additional ones (Ω_{mo} and Ω_{ro}) in the standard model. Most important though is the linear dependence of H on z in our model. As will be clear in the next section, the absence of adjustable parameters will be seen to have very important implications for processing and evaluating both the data and our theory.

6.3 Relation of distance to z

Before considering the Hubble data, let's note one other useful relationship that follows from the considerations above, namely the relation between z and distance away of an astronomical body, say D. Since $H(t_s) = 1/t_s$ and $H(t_o) = 1/t_o$, equation 76 can be rewritten in the following way:

$$\frac{1}{t_s} - \frac{1}{t_o} = \frac{z}{t_o} \tag{78}$$

where t_o is the present time and t_s is the time light was emitted. Or solving for t_s/t_o:

$$\frac{t_s}{t_o} = \frac{1}{1+z} \tag{79}$$

But the distance away is $D = c\,[t_o - t_s]$, or expressed in terms of z:

$$D \;=\; c\,t_o \left[\frac{z}{1+z}\right] \;=\; R_o \left[\frac{z}{1+z}\right] \tag{80}$$

Our result that $D/R_o = z/(1+z)$ can be contrasted with the prevailing model given by [24] as:

$$\frac{D\,H_o}{c} \tag{81}$$
$$= (1+z)\ |\Omega_k|^{-1/2}\ sinn\left\{|\Omega_k|^{1/2}\int_0^z [(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda]^{-1/2}\right\}$$

where $\Omega_k = 1 - \Omega_M - \Omega_\Lambda$, and *sinn* is sinh for $\Omega_k \geq 0$ and sin for $\Omega_k \leq 0$. The differences between the theories will prove to be crucial when we consider the supernovae data in Section 7.3 below.

6.4 $\tilde{H}(z)$ versus z from the data

The recent highly cited paper by Yu et al. [4] contains a thorough analysis of the Hubble 'constant' evaluation from 36 sources. Most conveniently (for us at least), their Table 1 contains the data summarized as $\tilde{H}(z)$ versus z where z is defined by equation 69 for values of $0.07 \leq z \leq 2.36$ and $\tilde{H}$ [km/s/Mpc] values ranging from 68.6 to 227 (along with rms error estimates). They carry out an extensive evaluation of the standard model, so we will focus instead only on comparison of their data with our new result equation 76.

The top part of Figure 1 plots our theoretical curve of equation 76 together with the measured $\tilde{H}(z)$ versus z-values of Yu et al. [4]. The error bars are from the Yu et al. paper as well. The solid line on the top figure corresponds to $H_o = 63.6$ km/s/Mpc, our optimal fit to the data. Our value of $H_o = 63.6$ km/s/Mpc) corresponds to $t_0 = 15.4 \times 10^9$ years (15.4 billion years).

The bottom figure shows the same optimal fit plus addition popular values: the dashed lines correspond to $H_o = 61,\ 67$ and 72 km/s/Mpc's. 72 is clearly too large, and 61 is too small. Either 67 and 63.6 would be acceptable choices. The mean square relative error of our fit is 9%; but the same error for 67 is only a few percent larger. The scatter in the figures appears to be randomly distributed, and both the 64 and 67 theoretical curves lie within the error bars of all but two of the data. A few outliers are largely responsible for the RMS relative error.

The only adjustable parameter in our theory is H_o. The best fit value of 63.6 km/sec/Mpc is well within the error bounds of the two values of 63.8 ± 1.3 and 65.2 ± 1.3 inferred by Riess et al. [15]. Rounding it off to $H_o = 64$ km/s/Mpc corresponds exactly the value deduced from gravitational lensing by SN Refsdahl by Vega-Ferrero et al. [25].

A different data set might produce a different value of H_o. So also shown on the plot is the theoretical curve using the recently popular Planck value of $H_o = 67$ for which the RMS relative error of the fit increases only to 10%.[3] The random error varies inversely with the square root of the number of independent estimates (only 36 in the present case), so the error bounds will surely drop as more data are acquired, especially as new theoretical considerations are included in the analysis and old ones winnowed out. This will be especially true if astronomers start presenting their data in the form of Yu et al. [4] instead of averaging it all together to get a single averaged value of H.

Clearly no matter the exact value of H_o, the theoretical equation 76 is quite satisfactory. While $H_o \approx 67$ works fairly well, the proposed value of 15.4 billion years certainly resolves any issues about whether there could be stars older than the currently proposed age of the universe of 13.8 billion years. The present estimate for the 'Methuselah' star (140283) of 14.46 ± 0.8 Gyr by Bond et al. [26] places it marginally within the previous age estimates, but well within the 15.4 Gyr derived above. We will use $H_o = 63.6$ km/s/Mpc

[3] Note that in Section 6.5 we use our value of H_o with equation 76 to account for time and scale of the Cosmic Background Radiation.

in the remainder of this paper unless otherwise specified.

6.5 Cosmic Microwave Background Radiation Fluctuations at $z = 1100$

The cosmic background radiation (CMR) is of interest for several reason. first, This data has been used to infer the H_o = 67km/s/Mpc that was cited in the preceding section, Second, the cosmic background radiation is generally believed to be the footprint of the universe at or about the time that photons could propagate [16], or when the temperature was about 3000 degrees K.

The average temperature is uniquely determined by the radiation spectral peak, say,

$$\lambda_m = b/T_u \tag{82}$$

where T_u is the absolute temperature of the universe and $b = 2.90 \times 10^{-3}$ m-K is the Wien's displacement constant. By the same arguments put forth in Section 5 above, we can conclude that $\lambda_m(t) \propto \delta(t)$. It follows immediately that at any time, t, the average absolute temperature of the universe divided by the average temperature now is given by:

$$\frac{T_u(t)}{T_u(t_o)} = \frac{\delta(t_o)}{\delta(t)} = \frac{t_0}{t} \tag{83}$$

$$= 1 + z \tag{84}$$

where the dependence on z, equation 84 follows from equation 79. Note that the z-dependence is the same as for the standard model, but the time dependence is not. Also note that $T_u(t)/T_u(t_o)$ could equally well have been expressed using τ as the variable as noted in Section 2 above, in which case the ratios would be $\exp(\tau_0 - \tau)$.

From the Planck estimate that the temperature now is 2.725 K, we can estimate that we should be able to see back to only $t = 14$ million years. Note that this is exactly the value we obtain by inserting $z = 1100$ into equation 79. This is substantially greater than the 380,000 years after creation usually cited. But it is actually farther back than the FLRW estimate, since our universe is over a billion years older.

Since the temperature value depends only on radiation spectra and time with no other assumptions, it is really not affected by the other difficulties of measurement. So if we have correctly interpreted the Hubble data, or it doesn't change from the values we used, this number might be quite accurate.

Now how about the scale of the CMB fluctuations? In our model universe, any initial inhomogeneities would now appear re-scaled by $\delta(t)/\delta(t_*)$ where t_* would be any time shortly after the non-linear processes dominate, and surely closely related to the time estimated below for the Quantum Field Theory energy input. So our theory should be able to predict (or at least account for) the present scale of the inhomogeneities, say L_{CBR}. If we use an estimate of the CMB length scale to be $L_{CMB} \approx 2.5 \times 10^{24}$ m (or 1 degree), the ratio to the Planck length, L_P, is $L_{CMB}/L_P \approx 1.6 \times 10^{59}$, again clearly indicating its early quantum origins. This points to a virtual origin of $-(1/0.16)\, t_P \approx -6.3\, t_P$. This is the same order of magnitude as the $-10.5\, t_P$ estimated below in Section 8.4 using the QFT energy estimate. Either is perfectly consistent with the time for the non-linear physics to transition from quantum mechanics to our 'turbulence similarity' type behavior.

7 Astronomical distance vs z using type Ia supernovae

One of the greatest challenges of astronomy has been to measure distance, especially outside of our own galaxy. In this section we explore the consequences for our theory on the interpretations of recent measurements, especially type Ia supernovae.

We have already considered how blackbody radiation would propagate in our new $\tau, \vec{\eta}$-universe, and in particular how any spectrum would be modified. And we have considered the consequences of an inverse square law in our expanding coordinates. Now we will apply this information to review the methodology and terminology usually used to compute distances from intensity measurements of a single astronomical body. And we will examine the results of applying our new theory to it. Finally we consider the recent data of [27, 15, 5, 6]. These are the data that have been used with a $1+z$ multiplication factor to argue that the universe is expanding at an increasing rate. We note that Love and Love [28] have observed empirically that without the $1+z$ factor in the supernovae data, the universe is not accelerating. Our theory will be seen to provide excellent agreement to the data *without* the $1+z$ corrections. The next section uses our results from Section 5 about how blackbody radiation propagates, and finds no reason for such corrections.

7.1 The inverse square law

The traditional way to use the inverse square law in astronomy has been to express distances in Megaparsecs using $m - M$ where m is defined as the apparent magnitude and M the absolute magnitude (v. [29]). m and M are essentially logarithms of the measured intensities and are usually compared to a reference value using the following formula:

$$m - m_{ref} = 2.5\ log_{10}\ I/I_{ref} \tag{85}$$

where I is the measured intensity and I_{ref} is a convenient reference. And m_{ref} is chosen as the value m would have, say M, if the same star were at a distance of 10 parsecs. So if the intensity is presumed to be inversely proportional to distance squared, say $I \propto 1/D^2$, this formula reduces to

$$\mu = m - M = 5\ log_{10}\ D + 25 \tag{86}$$

where D is measured in Mps. Corrections exist for a variety of conditions including cosmic dust, expansion of the universe, etc.

7.2 Relation of $m - M$ to redshift parameter z

Given that our inverse square law can be expressed in BOTH distance, D (in $t, \vec{x}$-space), AND normalized distance $D/\delta(t)$ (in $\tau, \vec{\eta}$-space), we must be very carefully about applying corrections from one frame to another. We can not just blindly use equation 86, but we must return to the definition of equation 85. Applying this yields:

$$m - m_{ref} \ = \ 2.5\ log_{10} \left\{ \frac{[D_{ref}\ /\ \delta(t_{ref})]^2}{[D\ /\ \delta(t_o)]^2} \right\} \tag{87}$$

where t_o is the present time and t_{ref} is the reference time. BUT these are the same, so the δ's cancel out! So our expression above reduces to:

$$m - m_{ref} \ = \ 5\ log_{10} \left\{ \frac{D_{ref}}{D} \right\}, \tag{88}$$

exactly (as expected) the result we would have gotten had we started in $t, \vec{x}$-space . BUT this is not as trivial a result as it seems, since we had to reference BOTH D and D_{ref} to their own space, NOT just one of them. Referring only D (and not D_{ref}) as well is the origin of the oft-used $1+z$-correction. Clearly any correction for expanding space of just one of them would be incorrect.

Now we know from equation 80 that $D = R_o\ z/(1+z)$ where $R_o = c\ t_o$. Substitution yields:

$$
\begin{aligned}
m - m_{ref} &= 5\ log_{10} \left\{ \frac{D_{ref}}{R_o\ z/(1+z)} \right\} \\
&= -5\ log_{10} \left(\frac{z}{1+z} \right) \ +\ 5\ log_{10} \left(\frac{D_{ref}}{R_o} \right) \qquad (89)
\end{aligned}
$$

If we choose D_{ref} to be 10 pc, and express $R_o = c\ t_o = c/H_o$, it follows immediately from equations 86 that:

$$
\mu = m - M = -5\ log_{10} \left[\frac{z}{1+z} \right] \ -\ 5\ log_{10} \left[\frac{c}{H_o} \right] + 25 \qquad (90)
$$

where c/H_o must be expressed in Mpc. Note that the only parameter in this equation is H_o, the present value of the Hubble parameter and the inverse of the age of the universe. No further correction of data or theory for expansion of the universe is necessary, thanks to the fact that the definition of equation 86 uses only relative values.

For the value of $H_o = 63.6$ km/s/Mpc derived in Section 6.4 above using the data of Yu et al. (or $t_o = 15.4 \times 10^9$ years), equation 90 reduces to:

$$
m - M = -5\ log_{10} \left[\frac{z}{1+z} \right] \ -\ 43.4 \qquad (91)
$$

This simple result can be contrasted with the much more complicated expression that results from substituting the standard model result of equation 81 for D into equation 86. Here the only two input data are H_o and M, both of which can be obtained independently by theory and measurement.

7.3 Comparison of theory with data for $m - M$

The first extensive use of the type Ia supernovae data was that compiled in the Calán-Tololo database by Hamuy et al. [30, 31], but only for relatively small values of z ($0.03 < z < 0.10$). This was extended to much larger values up to $z = 0.83$ by Riess et al. [15] and Perlmutter et al. [5]. There have been many papers since in more or less agreement, but we include only the additional results of Knop et al. [6]. All of these presented an extensive evaluation of their distance measuring methodologies. All presented their data in a variety of tables (including the Calán-Tololo data), and show in detail how they corrected the data. Most problematic (at least from the perspective of this paper) is the following quote from the Perlmutter et al. paper:

> "For the supernovae discussed in this paper, the template must be time-dilated by a factor 1 + z before fitting to the observed light curves to account for the cosmological lengthening of the supernova timescale. (Goldhaber et al. 1995 ; Leibundgut et al. 1996a ; Riess et al. 1997a)."

This is of course consistent with our arguments in Section 5.3 above, but they unnecessarily compensate for a growing universe by multiplying their data (column d by $1 + z$). Therefore we choose instead to use their original data and that directly measured (column b)).

Figure 2 shows the data from all four groups as summarized columns 2 and 4 of Table 1 and Table 2 of [5], and columns 2 and 3 of [6]. For both papers, column 2 is z, the redshift. Column 4 of [5] is plotted as black squares. Column 3 of [6] is plotted as red circles. The blue diamonds are the Calán-Tololo data of [30, 31] as collected in Table 2 of [5]. We have NOT used the 'corrected' data which includes the multiplication by $1 + z$.

The top figure shows the data with the theoretical curve computed from equation 91 using a value of the absolute magnitude $M = -18.5$. The lower figure shows three values of the theoretical curve computed from equation 91 using values of the absolute magnitude $M = -18.0$, -18.5 and -19.0. The -18.5 value provides an excellent fit for all values of z. And the two other values pretty much bound most of the data. Since each star presumably has its own slightly different value of M, there is no reason a single value of M should fit all the data. The fact that it does so even approximately is consistent with observations that all supernovae of type SN Ia seem to have absolute magnitudes that fall in a narrow band from about -18.5 to -19.5 (c.f. [29]), consistent with a Chandrasekhar limit. So the scatter is quite likely due to individual variations of M. Regardless the agreement of all three sets of data suggest strongly that no special correction for $1 + z$ is necessary, exactly as argued in section 5 above. This is consistent with the observation above in Section 5 that it is the entire spectrum (and its integral) that is shifted, not just individual frequencies, so no such correction is necessary.

Before leaving this section, we note that our theory assumes a flat universe, and it is slowly decelerating as $-1/t^2$. We see nothing in this data that suggests otherwise. If, however, we had left the $1+z$ factor included by [15, 5] and others, the opposite conclusion could have been reached. Our theory can be made to fit that data as well by choosing value of M between about -17.5 to -18.5 with a best fit curve corresponding to $M = -18$. The price paid however is that there is a sharp discontinuity with the Calán-Tololo data, which falls well below any of these curves. Therefore given our physical arguments for NOT including the $1+z$ factor (as appears to have been common practice), the continuity from one data set to another of our treatment of the data, and the complete absence of any obvious new physics to explain a shift, we stand by our analysis and conclusions.

Riess et al. and Perlmutter et al. [15, 32, 5], fit the standard model result of equation 81 to their data pre-multiplied by $1+z$ with $\Omega_M, \Omega_\Lambda = 0.5, 0.5$. This in addition to the choice of M is a three-parameter fit. It is clear from the form of equation 81 why they needed the $1+z$-prefactor to have any success at all with the fit (and why they might have convinced themselves that it was necessary). Our theory fits their original "uncorrected" data quite nicely with only a choice of M which is well within the bounds of most estimates. In fact, the scatter might be largely due to slightly different values of M for each star.

8 Mass and Energy

Now we get to the crux of this paper. We have already seen that our proposed universe can describe the kinematics that have been observed and published. Now we must investigate the underlying dynamics.

Also, do we still need dark energy and dark matter? Surely there have been difficulties identifying both in nature, and the need can only be inferred indirectly. Also as our observations above in the two preceding sections make clear, neither is necessary to account for the data usually used to justify the indirect inferences. Hence the alternative presented by our theory might be very attractive.

8.1 Implications of the similarity solution

Since our presumed model is Minkowski in $\tau, \vec{\eta}$-space, it must be in all [13]. As noted earlier, this does not imply that the metric tensor, $g_{\mu\nu}$, in other spaces is $[-1, 1, 1, 1]$ nor that the Christoffel symbols will be zero. But it does dictate that the Riemann tensor, $R_{\mu\nu\alpha\beta}$, Ricci tensor, $R_{\mu\nu}$, and Ricci scalar, R, will all be zero.

Thus the field equations (equation 5) in contravariant form reduce to:

$$0 \; = \; \left[T^{\mu\nu} - \frac{1}{2} T \; g^{\mu\nu}\right] \tag{92}$$

where we have dropped the pre-factor of the right-hand-side since the left-hand-side is zero. So for our postulated universe:

$$T^{\mu\nu} = \frac{1}{2} T g^{\mu\nu} \tag{93}$$

where T is the invariant defined by equation 6. Note that our $T^{\mu\nu}$ differs significantly from that used in previous analyses, since we have deliberately relaxed the usually applied condition that its divergence be zero (c.f. equation 4.15 in [13]). Thus we have not *required* that energy be conserved. Our reasons for doing so are several (and discussed below), but the most obvious is that theories which do assume conservation of energy do not seem to have been successful, at least without inventing mysterious and as yet unobserved energy sources. We will have more to say about this later. Because of assumed spatial homogeneity, T can at most be a function of τ in $\tau, \vec{\eta}$ coordinates.

It has been traditional to assume that $R^{\mu\nu} = 0$ implies that $T^{\mu\nu} = 0$, and thus describes only empty space. But given our assumptions (or rather lack of one) about the divergence of $T^{\mu\nu}$, $R^{\mu\nu} = 0$ only implies that space is flat, and that $T^{\mu\nu} = T \; g^{\mu\nu}/2$, not that either are zero. Also we note that the pre-factor of $8\pi G/c^4$ completely disappears. Since the left-hand-side of equation 92 is zero, there is no counterpart to the usual criterion for a critical density. In our theory equation 4 completely vanishes, and so there is no critical density.

Alternatively, equation 5 could equivalently have been formulated using the more usual form of the field equations given as ([13]):

$$R^{\mu\nu} + \frac{1}{2} \; R \; g^{\mu\nu} + \Lambda \; g^{\mu\nu} = \left[\frac{8 \; \pi \; G}{c^4}\right] \; T^{\mu\nu} \tag{94}$$

where R is the Ricci scalar and we have separated out Einstein's cosmological constant Λ. Clearly if $R^{\mu\nu}$ and R are identically zero and T is non-zero, then Λ is non-zero; and vice-versa. We have chosen to proceed using equation 5 since it seems less contrived than a 'cosmological constant' (or cosmological function in this case), which even Einstein himself thought contrived.

8.2 What is T?

We note that dimensionally there are only two independent parameters in the governing equations: G and c. In the absence of curved space ($\kappa_U = 0$, our flat space hypothesis), there is no imposed length scale. The Planck scales introduce G along with Planck's constant and c, to characterize when gravitational effect become important. But after the initial period when $\delta >> L_p$, where L_P is the Planck length scale, the Planck length is far too small to be relevant. The evolution from quantum mechanics through this period has been described by Gibson [33, 34, 35]) as an inverse turbulence cascade with ever increasing scales of motion. So the Planck parameters become ever relatively smaller and smaller, and *no new parameters enter*. And there is no combination of the two parameters, G and c, alone that can produce a length scale or a time scale, nor anything with the dimensions of energy or mass densities. So any such scale must arise from the equations themselves or assumptions about them. A spatial coordinate could provide the missing parameter, but not in a homogeneous environment. So a length scale dependent only on τ (or t) must exist to provide the missing parameter. The only choice is, of course, the length scale, $\tilde{\delta}(\tau)$ (or $\delta(t)$), which we introduced at the beginning.

Since the left-hand-side of equation 92 is zero, we have some freedom in choosing how to scale the right-hand-side. But we stick with convention and give the contravariant tensor, $T^{\mu'\nu'}$, in physical coordinates the dimensions of energy density; i.e. the same as ρc^2 where ρ has dimensions of mass per unit volume. So since T is given by equation 6 and the contravariant metric has dimensions of length squared, the appropriate choice for T itself is:

$$T \;=\; -\,[2\ G_*]\frac{c^4}{G\ \tilde{\delta}^4(\tau)} \;=\; -\,[2\ G_*]\frac{c^4}{G\ \delta^4(t)}. \tag{95}$$

where $[2\ G_*]$ is an unknown coefficient. Note that we have used the freedom presented by the zero left-hand-side of equation 92 to choose the constant of proportionality as $[2\ G_*]$. G_* will be related to the energy and mass densities in equation 100 and 101 below. We note that equation 95 can be expressed using the determinant, g, of the contravariant metric tensor of equation 11:

$$T \;=\; -\,[2\ G_*]\frac{c^4}{G}\ \sqrt{-g}. \tag{96}$$

If integrated over a volume, this is exactly the form of the Einstein-Hilbert action, but for the scalar invariant T instead of the Ricci scalar R (v. [13].)

Since we know $\delta(t) \;=\; c\ t$ and $\tilde{\delta} = t_1 \exp\tau$, we can rewrite equation 95 in terms of t and τ as:

$$T \;=\; -\,\frac{[2\ G_*]}{G\ t^4} \;=\; -\,\frac{[2\ G_*]}{G\ t_1^4}\ \exp\left(-4\ \tau\right). \tag{97}$$

The time derivative of T is clearly non-zero, meaning that the divergence of $T^{\mu\nu}$ is non-zero, consistent with our observation about it at the beginning of this section.

Since t_1 is on the order of the Planck time ($t_P = 5.4 \times 10^{-44}$ s), $1/t_1^2$ is a very large quantity indeed. Recall that t_1 is the lower limit of the integral in equation 8, and corresponds to $\tau = 0$. In Section 9 we will identify both of these unambiguously with the time of the Big Bang, or at least a virtual origin corresponding to it. In the next section we shall show how to express T in terms of the time-dependent mass density, ρ, and the rest mass energy per unit volume, e.

8.3 Mass density and rest mass energy

Clearly the behavior of mass density, ρ, and rest mass energy, e, with time is crucial to understanding the expansion of our universe. Earlier FLRW theories *assumed* conservation of mass and energy. This would require (using our symbols) that mass density, say $\rho\ \delta^3$, and rest mass energy, say $e\ \delta^3$, to be constant. Abundant observations suggest strongly that they cannot be maintained as constants without a large source of external (dark) energy. This is the major point of departure of our theory from the classical theory. For our proposed universe, there is no reason to believe that either mass or energy are conserved, because we are allowing physical time to be measured in non-equal increments.

One way to see the implications of this is the following simple example. Consider the acceleration of a a simple lump of matter by a force applied to it. Since mass is the ratio of the applied force to the acceleration, if the acceleration is measured differently using non-equal time increments, then the mass must be defined differently. Similar conclusions apply to the kinetic energy.

Actually the dimensional argument for ρ and e requires a bit more subtlety and physical insight than we gave T in the preceding subsection. Both quantities could have been influenced by things that added to them outside the region where c, G and $\delta(t)$ (or t) were the only parameters (like quantum effects in an earlier era). So the most we can say on dimensional and physical grounds alone is that *changes in the density*, say $\Delta\rho$, and *changes in the rest mass energy*, say Δe, are given by:

$$\frac{G\ \Delta\rho\ \delta^2}{c^2} = G\ \Delta\rho\ t^2 = G_* \tag{98}$$

$$\frac{G\ \Delta e\ \delta^2}{c^4} = \frac{G\ \Delta e\ t^2}{c^2} = G_* \tag{99}$$

where G_* is an absolute constant set by the initial conditions. Note that $\Delta\rho$ and Δe are functions of time t only, a direct consequence of our assumed spatial homogeniety. In fact, $\rho \to 0$ in the limit as $t \to \infty$ is the only reasonable choice; and this in turn implies that $\Delta\rho = \rho$ and $\Delta e = e$. So these reduce simply to:

$$\frac{G\ \rho\ \delta^2}{c^2} = G\ \rho\ t^2 = G_* \tag{100}$$

$$\frac{G\ e\ \delta^2}{c^4} = \frac{G\ e\ t^2}{c^2} = G_* \tag{101}$$

where G_* is a universal constant.

We noted in the preceding section that we could express the stress-energy scalar, T, in terms of ρ or e. It follows directly on dimensional grounds and using equations 95, 100 and 101 that:

$$T = -2\ \frac{\rho\ c^2}{\delta^2}\ \ =\ \ -\ 2\ \ \frac{e}{\delta^2} \tag{102}$$

It is easy to show that our choices of sign and numerical coefficient for T insure that:

$$T^{0'0'} \;=\; \frac{1}{2}\, T\; g^{0'0'} \;=\; \rho\, c^2 \;=\; e. \qquad (103)$$

since $g^{0'0'} = -\delta^2$.

To conclude this section we note that the idea that energy is *not* conserved is really not a new idea, especially since the work of Noether [36] which makes clear the relation between energy and the choice of time. And it has been noted before that a proper definition of inertial mass depends on how we define time [37, 38]. Also, Carroll [13] observed the following which although in a different context appears quite relevant here:

> Clearly in an expanding universe, the energy-momentum tensor is defined on a background that is changing with time; therefore there is no reason to believe that energy should be conserved.

He then goes on to argue that the zero divergence of $T^{\mu\nu}$ implies there is a law in which something is conserved. As noted above, our $T^{\mu\nu}$ does not have zero divergence, but our $T^{\mu\nu} - (T/2)\, g^{\mu\nu}$ does. And as Carroll suggests, there is a corresponding conserved quantity: $G_* \;=\; \rho\, G\, \delta^2/c^2 \;=\; e\, G\, \delta^2/c^4$.

8.4 The 'Worst Prediction in the History of Physics'

There is a direct relation between Einstein's Field equations and the equations of turbulent flow in fluid mechanics. Our solution to the field equations with its time-dependent length scale is in fact a close analog of similarity solutions of the averaged Navier-Stokes equations for homogeneous turbulence (c. f. [17, 18]). Gibson [34] has argued that these post-quantum and post-inflation early processes would have been similar to an inverse Kolmogorov cascade in turbulence where the scales grow and energy is moved to larger and larger scales. Such high (or infinite) Reynolds number 'turbulence' can be characterized in spatial Fourier space by a wavenumber energy spectrum that rises quickly to a peak at roughly the inverse of the characteristic length scale, say like our $\delta(t)$, then rolls of as $k^{-5/3}$ where k is the magnitude of a spatial wavenumber vector. The result is that the energy is spread over a band roughly characterized by wavenumber equal to $1/\delta$ (c.f. [39] or any book on turbulence). Alternatively the field could be described in terms of its evolution in time, usually a power law determined by the initial condition, exactly as we have in equations 100 to 101.

So the question can be asked: How far back in time can our theory be expected to apply (assuming it is valid at all)? We show in this section that it in fact appears to reasonably describe everything from Planck times to the present time. And it does so without dark matter or energy. This is quite different from the popular view at the moment. First we consider the Quantum Field Theory estimates of the energy put in at the beginning. And then we compare that to the best estimates of the visible mass in the universe.

One of the most unsuccessful theoretical results of the past few decades has been the large discrepancy between the predictions of Quantum Field Theory and the observed energy density in the cosmos. The ratio of the QFT prediction, 10^{72} GeV/m^3 or 10^{111}J/m^3, to the observed mass (or energy) is usually estimated at 10^{120}. This has been described as the '*Worst Prediction in the History of Physics*' [13, 19, 40]. (Caroll and Ostlie [21] in their section on 'The Early Universe" provide an nice summary of how this result was obtained from the uncertainty principle.) A modification to the QFT calculation to include Lorentz invariance [41] reduces this discrepancy by a factor of 10^{60}, which is still huge. This is all the more troubling since QFT seems to accurately predict the magnetic dipole moment. These results have been interpreted a number of ways and have been dubbed 'vacuum energy'. We interpret them here as an initial condition.

The search for how much matter there is in the universe has been detailed in many publications including popular articles and all textbooks. The basic problem has been that the mass density inferred from visible matter is much less than the critical density of equation 3. There have been numerous studies in the last decade using various methods of processing both astronomical observations and simulations to try to estimate the visible (and invisible) matter in the universe. Table 2 of the very recent paper of Abdullah et al. [2] lists the results of 20 different extensive studies. Their estimates of the mass density of matter in the universe range from a low of 0.22 ρ_c to 0.40 ρ_c, where ρ_c is the critical density defined by equation 3. The average of all 20 estimates was approximately 0.29 ρ_c. Their value of ρ_c was calculated from equation 3 using the age of the universe as 13.8 billion years and various values of the Hubble parameter.

The most recent value cited in the table was that of Abudullah et al. themselves for which the present mass density, ρ_o, was estimated to be 0.305 $\rho_c \pm 0.04$ ($\Omega_m = 0.305$), which gives (using their parameters): $\rho_o = 2.72 \times 10^{-27}$ kg/m^3.

Our theory, equation 100, says that changes in mass or energy density should vary inversely with time-squared; i.e.,

$$\frac{\rho(t_{QFT})}{\rho(t_o)} = \left[\frac{t_o}{t_{QFT}}\right]^2 \tag{104}$$

We don't know the value of t_{QFT}, but we can certainly calculate it by working backwards from the Abduallah et al. value and the two QFT estimates. The QFT estimates can be converted from energy to mass by dividing by c^2, and respectively give $\rho_{QFT1} = 1.11 \times 10^{94}$ kg/m^3 for the original estimate, and 1.05×10^{47} kg/m^3 for the revised Lorentz invariant estimate. We will use our value for t_o estimated in Section 6.4 as 15.4 billion years (or 4.9×10^{17} seconds). From the original QFT estimate we calculate $t_{QFT1} = 2.4 \times 10^{-43}$ seconds, or about 4.5 Planck times. For the second, $t_{QFT2} = 7.8 \times 10^{-20}$ seconds or about 1.5×10^{24} Planck times. The first corresponds to the beginning of the importance of gravitational effects; the second is well into the grand unification epoch (v. Table 1 below). Both QFT estimates are well before the age of galaxies or even before photons can propagate. Most importantly, both are close enough to $t = 0$ to reasonably associate them with the Big Bang. And both were obtained with **no adjustable constants** using only our theory and the Abudullah et al. estimate of the current observable mass density of the universe.

Figure 3 shows a plot of ρ/ρ_o versus t/t_o from the Planck time t_P to the present t_o using equation 100. The data have been normalized by $\rho_o = 2.72 \times 10^{-27}$ kg/m^3, the Abdullah et al. value. The two Quantum Field Theory (QFT) estimates are shown by the orange diamonds. The present value is the blue triangle.

Note that no particular 'time' is given for when these QFT estimates should be applied, so we extrapolated back from the present. We know, however, that the functional form of their time dependence is the same as our theory (see discussion in Section 8.6 below and [13])). So if we had a beginning time we could have connected the curves instead of showing the QFT results as just points. This would have alleviated our need to infer the present density value from just measurements, and provided a useful comparison whether the Abdullah et al. estimate is large enough.

Regardless of our inability to pin down the exact relation, the results must be regarded as a spectacular result and success – especially for much maligned quantum field theorists, the astronomers and our theory! We will have more to say about this below, but for now it is clear that our theory is treating the universe as an initial value problem. All of the energy (and mass) is added at the beginning. And it is simply being dispersed – outward in physical coordinates. We can now with some confidence use the Abdullah et al. value to calculate the value of G_* in equations 100 and 101. The result is:

$$G_* = \rho_o \; G \; t_o^2 \;=\; 0.043 \qquad (105)$$

Different astronomical results could change this result.

8.5 The stress-energy tensor revisited

We have all the pieces now to examine in detail the stress energy tensor. In $\tau, \vec{\eta}$ coordinates it is simply:

$$T^{\mu\nu} \;=\; \frac{T}{2}[-1,1,1,1] \;=\; -\frac{[G_*]}{G\; t_1^4}[-1,1,1,1]\; \exp\left(-4\;\tau\right) \qquad (106)$$

where we have substituted equation 97 for T. We will associate t_1 with the time of energy input below.

We note that this is similar in form to the Quantum Field Theory result of Carroll [13] in Chapter 9 when his equations 9.166 and 9.133 are combined. There he considers a two-dimensional flat Minkowski space of infinite extent, but governed by both Einstein's equations and quantum mechanics. Even his time is logarithmic. Surely dimensional analysis alone dictates the physical time dependence in both solutions. But we offer two possibilities in Section 8.6 below which considers radiation as well.

More similarities to the QFT theory can be seen by examining the $t, \vec{x}$ form of our contravariant stress energy tensor, $T^{\mu'\nu'}$. Combining the equation 102 with equations 93 yields:

$$T^{\mu'\nu'} = \left\{ \begin{matrix} \rho c^2 & \rho u c & \rho v c & \rho v c \\ \rho u c & p + \rho u^2 & \rho u v & \rho u w \\ \rho u c & \rho u v & p + \rho v^2 & \rho v w \\ \rho u c & \rho u w & \rho v w & p + \rho w^2 \end{matrix} \right\} \qquad (107)$$

where we have used equation 30 to obtain the contravariant velocity components, and the De Sitter space equation of state:

$$p = -\rho\, c^2. \tag{108}$$

This pressure is not 'pressure' in the usual sense, but the mean normal 'stresses' of the averaged motion – like the Reynolds normal 'stresses' in classic fluid mechanics turbulence. Note that there is a slight difference from the usual equation of state (equation 4.33 in [13]) because we have normalized differently.

Aside from the normalization these equations are exactly the form of Einstein's stress energy tensor Carroll chose. The $0'0'$-component is the rest mass energy. And the diagonals are the pressure and longitudinal momentum fluxes. The off-diagonals are 'Reynolds-stresses' and disappear if the matrix is diagonalized or cast in spherical coordinates.

It is interesting to discuss the role of pressure here. In $\tau, \vec{\eta}$-space, the pressure is everywhere the same and there is no motion, both by hypothesis and the assumed homogeniety. The $t, \vec{x}$-space is far more interesting. It is not homogeneous, since looking out in space is really looking back in time. But (on the average) it looks the same to all observers no matter their location. And no matter where they sit in it, the universe appears to be accelerating away from them since $u_r \propto r$ at a single instant in time. But what is driving this 'apparent' acceleration? Since the density is decreasing with time and $\rho c^2 \geq 0$, the pressure $p = -\rho c^2$ must be increasing in time from a deep negative value. But the p at any radius r is the p that was really the pressure at an earlier time, and that was even more negative than it is at present at the location of the observer. So the pressure gradient is negative; i. e. decreasing with increasing r. Thus to an observer in $t, \vec{x}$-space, this negative pressure gradient appears to be accelerating the flow away, and driving a positive momentum flux outward. Most importantly, there is no need for additional sources of pressure, momentum, matter or energy; i.e., especially dark matter or dark energy. The only necessary contribution to obtain what we think we see was put there at the beginning.

Another interesting observation: our solution looks like a three-dimensional analog of Carroll's two-dimensional QFT description in Chapter 9 Section 5. While Carroll's solution and ours began from opposite ends of the cosmic evolution, clearly the assumption of homogeneous Minkowski space dominates the behavior. In fact, Section 9.5 of [13] is almost exactly our solution if you interchange his $t, \vec{x}$-coordinates with his $\tau, \vec{\eta}$-coordinates. His 'Rindler coordinates' are our $t, \vec{x}$-coordinates. And with a bit of work his log time and exponential factor are the same as our log time and $\tilde{\delta}(\tau)$. His equation 9.140 is exactly what we try to argue in Section 9 below. Since there are not any new parameters that enter the problem, it would be a surprise if these solutions were not the same. Unfortunately we have not access to enough information to establish that. Perhaps others will.

Alternatively, any resemblance of the two theories could be just simple dimensional analysis, since G and c are the only parameters. Also note that our universe is infinite with many overlapping spheres of visible universes, depending upon where one is sitting. Carroll interprets his results as though there is a single spherical one, but this seems to be a matter of interpretation, not mathematics which are consistent with an infinite one. The finite universe he describes leaves us needing inflation, while ours does not. It simply begins everywhere at the same time.

As the next section makes clear, radiation provides a possibility that the two theories might be different, each representing a different region of expansion – one very early, one very late. And separated by one which has at least one more parameter - radiation.

8.6 Radiation energy density

The preceding sections, and equations 100 and 101 in particular, show how the mass and energy densities vary with time. What they do not show is what portion is photons (or radiation) and what portion is matter. We will calculate the radiation part separately, and then identify the remainder as matter.

The radiation mass density is given by Freedman et al. [29] as:

$$\rho_{rad} \;=\; 4\,\frac{\sigma\;T_U^4}{c^3} \tag{109}$$

where $\sigma = 5.67 \times 10^{-8}$ W^{-1} m^{-2} K^{-4} is the Stefan-Boltzmann constant. As noted in Section 6.5, the present temperature is $T_{Uo} = 2.725$ K. This corresponds to a present energy density of 4.64×10^{-31} kg/m^3. Clearly this is negligible compared to the estimate 2.72×10^{-27} kg/m^3 which we used for the present density.

The contribution of photons above can be modified to include neutrinos. Nave[42] (see also Carroll and Ostlie [21]) suggests including a factor of 1.681 to account for neutrinos since the neutrino temperature dependence is also T_u^4 (but only below 3Mev $\approx 3 \times 10^8$K). This makes little difference to the contribution to the density at the present time. But it does affect where the radiation curve intersects our theory; i.e., the point before which only radiation is present.

Therefore before leaving this section we ask: When would the total mass in the universe have been due to radiation. By combining equations 109 and equation 83, we conclude that the radiation density as a function of time is given by:

$$\rho_{rad}(t) \;=\; 6.724\,\frac{\sigma}{c^3}\,\left[T_{Uo}\,\frac{t_o}{t}\right]^4. \tag{110}$$

where we have changed the factor of 4 to 6.724 in equation 109 to account for the neutrinos. Equating this to equation 100 yields:

$$6.724\frac{\sigma}{c^3}\,\left[T_U\,\frac{t_o}{t_r}\right]^4 = \frac{G_*}{G\,t_r^2}. \tag{111}$$

where we have defined t_r as the time they are equal. Solving for t_r yields:

$$\left[\frac{t_r}{t_o}\right]^2 = \frac{G\,t_o^2}{G_*}\,\left[\frac{6.724\,\sigma\,T_{Uo}^4}{c^3}\right]. \tag{112}$$

The right-hand-side is just the ratio of the two densities at time t_o; i.e.,

$$\left[\frac{t_r}{t_o}\right] \;=\; \sqrt{\frac{7.80\times 10^{-31}}{2.72\times 10^{-27}}} \approx 0.017. \tag{113}$$

So multiplying by 15.4 billion years implies that all of the energy was radiation energy at 262 million years after the beginning. Previous estimates were approximately 24,000 years, so this is a substantial difference. BUT it is still farther back in time than previously believed since our universe is older, in fact farther back in time than the 13.8 billion years often taken as the age of the universe.

Now here we are faced with a dilemma. Since the radiation density given by equation 110 is increasing more rapidly with decreasing time than equation 100, should we follow our theory back to the QFT estimate as we did in the preceding section? Or should we follow the radiation density values back to the QFT estimate? If we do the latter, the radiation densities equal the QFT results at $t = 4.5\times 10^{-14}$ seconds (8.3×10^{29} Planck times) and $t = 2.5\times 10^{-2}$ seconds (4.7×10^{41} Planck times) respectively, the latter time corresponding to the lower QFT value. While these are much later times, at least to those interested in this early period, it clearly makes no difference to the age of the universe. And both alternatives lead smoothly from our gravity-dominated theory to the quantum dominated era.

Figure 4 shows the same theoretical curve, equation 100, as in Figure 3 above, but only for $t/t_o \geq 10^{-4}$. Two radiation curves are shown corresponding to equations 109 and 110, the latter including the neutrino contribution. Also shown on the figure is the location in time of $z = 1,100$ (red arrow) where the temperature is 3000 degrees K, and the location in time of the Methuselah star (green arrow). If the arguments at the end of Section 8.5 are correct and the QFT and our solution are the same, the solid line should be followed all the way back until quantum mechanics dominates. The dashed line simply indicates where photons begin to be important (where the lines cross) and how they cease to dominate as baryonic matter is formed.

9 What does this all mean?

We have found a solution which seems consistent with many of the previously challenging astronomical observations and quantum field theory calculations. But we have entered this through the back door by assuming a similarity form and exploring the consequences. But exactly what equation has our solution solved? We suggest that our solution corresponds to the following tensor equation:

$$R_{\mu\nu} \;=\; R^{\alpha}_{\mu\alpha\nu} \;=\; I_{\mu\nu}\;\delta_D(\tau) \tag{114}$$

where $R_{\mu\nu}$ is the averaged Ricci tensor (including second-moments), $I_{\mu\nu} = [-I_*, I_*, I_*, I_*]$, and we have placed a Dirac delta-function, $\delta_D(\tau)$, on the right-hand-side. So $I_{\mu\nu}$ represents an impulse function of strength I_* at $\tau = 0$. Note that the averaged Ricci tensor in $\tau, \vec{\eta}$ -space is independent of $\vec{\eta}$ because of our spatial homogeniety assumption; and that $-\infty < \tau < \infty$. Also we have arbitrarily chosen $\tau = 0$ so that the time, t_1, the lower limit of equation 8, corresponds to the time of our impulse. So t_1 is the virtual origin and should be measured in Planck times.

The Riemann tensor itself is the product of two covariant derivatives, and it is highly non-linear (v. [12]). While it can be linearized for small amplitudes (like gravity waves), it most surely cannot be here since this impulse response is not small – it's causing the whole universe to expand. Even if we write out the averaged equations above and included all the second moment terms, we would still be left with the usual turbulence closure problem – more unknowns than equations. Now it may be possible to solve equations 114 without worrying about the higher order terms – like the inviscid flow solutions which work well for the aerodymics of bodies, for example. Or maybe appropriate closure approximations can be made, as in much of engineering turbulence. Surely numerical solutions should be possible, but from turbulence experience it is probably crucial to make the computational box at least an order of magnitude larger than $c\,t$ throughout the computation. (And as in turbulence computations, this should be a lot easier to achieve in our scaled coordinates.) We note that Carroll [13] succeeded in reducing his QFT equations to a wave equation (his equation 9.140), but we were unable to do that here. Perhaps we have missed the obvious.

Fortunately, as happens frequently in turbulence theory (especially for flows of infinite extent [43, 39, 18], the strong non-linearity has led to a limit cycle or attractor which in this case can be characterized by scaling time and space together. And our solution has revealed enough about its nature so that we can examine its transient behavior in another way. We can work backwards from what have already proven to be true.

We know that the mass density in $\tau, \vec{\eta}$-space is given by:

$$\tilde{\rho}(\tau) = \frac{c^2}{G\ \tilde{\delta}(\tau)^2} = \frac{c^2\ \exp(-2\ \tau)}{G\ \tilde{\delta}(0)^2} \tag{115}$$

where $\tilde{\delta}(0) = L_P \times (t_1/t_P)$ and L_P is the Planck length scale. It is easy to show that its derivative with respect to τ reduces to:

$$\frac{d}{d\tau}\tilde{\rho}(\tau) = -2\ \tilde{\rho}(\tau) \tag{116}$$

We can move the derivative term on the right-hand side of equation 116 to the left-hand-side to obtain the following differential equation:

$$\frac{d}{d\tau}\tilde{\rho}(\tau)\ +\ 2\ \tilde{\rho}(\tau)\ =\ I_{**}\ \delta_D(\tau) \tag{117}$$

where we have added to the right-hand-side a Dirac-delta function, $\delta_D(\tau)$ to represent an impulse function of strength I_{**} at time $\tau = 0$. Note that $\tau = 0$ (or $t = t_1$) is where our solution looking backwards in time thinks the impulse occurred, i. e., the 'virtual origin'. The reason for any difference from the time of an actual impulse is that our solution cannot account for physics it does not include – meaning all of the quantum mechanics. But it should merge smoothly to it, as we have assumed it does in the preceding sections.

The solution to equation 117 we already know, so it is not particularly interesting. But it is the relation of the impulse function and time that is of interest. If we integrate from $-\infty < \tau' \leq \tau$ and assume that $\tilde{\rho}(-\infty) = 0$, the result for $\tau > 0$ is:

$$\tilde{\rho}(\tau) \;=\; I_{**}\, e^{-2\,\tau} \tag{118}$$

So I_{**} is our Big Bang, and clearly should be associated with the Quantum Field Theory results as we did in the preceding section.

It is obvious then that at least our universe is an initial value problem. More importantly, there is no need for additional sources of energy or mass or whatever to sustain its continued expansion. It really was a Big Bang, and aside from G and c, nothing else matters.

10 Tools for future work by astronomers

The great solid mechanicist and experimentalist James C. Bell of the Johns Hopkins University often said to his students and collaborators:

> "Experimentalists sort theories." (J. C. Bell [44])

But as noted by [45], the problem often is that there is only one theory, so there is enormous pressure to prove it correct. R.R. Long [46] in a famous footnoted paper[4] remarked:

> Theoretical results accepted on the basis of very limited evidence, become after long periods of time impossible to overturn with even abundant contradictory evidence.

The problem of course is the lack of alternatives before ideas become cast in intellectual concrete. So at very least we have provided astronomers with another theory 'to sort'. And in this section we've tried to provide the tools to do so.

[4]The editors added a footnote saying they were publishing it in spite of the negative reviews because of the author's persistence over many years.

There have been numerous studies showing various methods of processing both astronomical observations and simulations to try to account for the visible (and invisible) matter in the universe. The recent paper of Abdullah et al. [2] and the earlier work of Poggianti et al. [3], for example, focus on clusters. These data on the number and mass of the galaxies in clusters are usually plotted as inferred mass or mass distribution versus z. We would like very much to have utilized these extensive results to evaluate our theory. For the most part we have largely failed, and were able to do so only for the cluster number dependence on z which are plotted in Figure 6 of Section 10.7 below. We suspect our failure largely represents our own short-comings (See the postscript to this paper for an explanation.) But also the masses inferred seem to have been unduly influenced by the need to conform to the Standard model. We simply are not qualified to judge. So in this section we offer a brief outline as to how our theory might be used in future studies. Or for re-processing existing data by those more qualified than us to do so.

10.1 Things look different looking back in time.

All of the conclusions of the preceding section were a consequence of spatial homogeneity of an infinite universe at a single time. But we can only look back in time. Our universe is assumed to be statistically homogeneous in space but *non-stationary in time, t*. So if we average over a particular shell of radius r, we are really looking at the average universe at time $t = (R_o - r)/c$. We examine below how our new theory affects our view of what we see.

Suppose we integrate the density of equation 100 over the entire visible universe, say $R_o = c\ t_o$. Looking away a distance r is the same as examining its state at time $t = (R_o - r)/c$. So the density is given at any distance, say $\tilde{\rho}(r)$ is given by:

$$\tilde{\rho}(r) = \rho(t_o - t) = \rho_o\ \frac{t_o^2}{t^2} = \frac{\rho_o\ R_o^2}{(R_o - r)^2}. \tag{119}$$

As noted above, this is only valid after the Planck time, t_P, or something proportional to it within a few orders of magnitude. Integrating this from $r = 0$ to the quantum limit $R_o[1 - \epsilon]$ where $\epsilon = c\ t_P/R_o$ yields the total observable mass in the 'visible' universe as:

$$total\ mass = 4\pi R_o^2 \rho_o \int_0^{R_o[1-\epsilon]} \frac{r^2}{(R_o - r)^2} dr \tag{120}$$
$$= M_o \left\{ \frac{1}{\epsilon} - 2ln\epsilon + \epsilon \right\}$$

where M_o is the actual mass in the visible universe at the present time, t_o; i.e.,

$$M_o = \frac{4\pi}{3} R_o^3 \, \rho_o \tag{121}$$

So the mass looking back in time is VERY much greater than the mass actually there at THIS time. Only the limit imposed by the Planck time (below some multiple of which our theory does not apply) saves us from it blowing up entirely. And even if divided by the volume of the visible universe, this is a very large number, and clearly is no way indicative of the average density at present. It is, however, indicative of the energy put in initially; in fact exactly the QFT energy discussed in Section 8.4.

Figure 5 shows how almost all of the contributions to this integral and integrand were in the earliest second, but are forever hidden to us behind a wall of invisibility by millions of years. We can only see the footprint in the cosmic background radiation.

Clearly we need a better idea. The basic idea will be to examine how the cumulative mass varies with radius (or z). The data of [2] and [3] show how the cumulative number of galaxies (or clusters of them) increases with radius starting at $z = 0.04$ where z is the red-shift parameter. We will try here to examine what our theory can contribute to understanding such measurements.

10.2 Variation of volume with z

For theoretical arguments it is easiest to place ourselves at the center and denote the distance away from us as r. So $r = R_o = c\, t_o$ would be the edge of our visible universe. But most measurements are recorded using the redshift parameter which is directly measurable. So before considering how the mass and cluster number dependencies vary with r and z it is useful consider how volume itself varies with z. We could simply define $\tilde{V}_r = 4\pi\, r^3/3$ and convert that to z if we knew the relation between z and r.

If we note that $R_o = c\ t_o$ and $R_o - r = c\ t$, it is easily established from equation 75 as:

$$\frac{r}{R_o} = \frac{z}{1+z} \tag{122}$$

So $z = 0$ corresponds to $r/R_o = 0$; and as $z \to \infty$, $r/R_o \to 1$ and we reach the edge of our visible universe.

It follow immediately that the volume as a function of z is given by:

$$V(z) = \tilde{V}(r(z)) = V_o\ \left[\frac{z}{1+z}\right]^3 \tag{123}$$

where we have defined $V_o = 4\pi R_o^3/3$. *Note that this is an actual volume, not a co-moving volume.* For $z << 1$, $V(z)$ varies as z-cubed. But the volume of the universe is approached quite slowly as $z \to \infty$. These differences will be seen to be quite important below.

10.3 Mass distribution as a function of r (or z)

It is clear from the divergent integral in equation 120 that what we really need is not a spatially (or temporally) averaged density but a cumulative density as a function of radius, r, or z. And then we can choose whether or not to normalize it with the cumulative volume or the current value of density, ρ_o, which hopefully we can determine by fitting the data.

By the same arguments that led to equation 120, the observable mass inside radius r should be given by:

$$\tilde{M}_g(r)\ \tilde{N}(r) = 4\pi \int_0^r \rho_r(r')\ r'^2\ dr' \tag{124}$$

where $\tilde{N}(r)$ is the cumulative number of clusters inside radius r, and $\tilde{M}_g(r)$ is their average mass (which can not be assume independent of radius since mass is not conserved). Note that the ϵ in our previous integral, equation 120, has become the running value of r/R_o here. We have tried to follow convention by considering separately the number of clusters and their mass, although this integral can only consider their product. Nor can it separately account for clouds of gas, so any such must be considered part of the same integral or treated as just another cluster.

Substituting for the density from equation 119 and integrating yields:

$$\frac{\tilde{M}_g(r)\ \tilde{N}(r)}{M_o} = 3\left[\frac{r/R_o}{1-r/R_o}\ +\ 2\ln(1-r/R_o)\ +\ r/R_o\right] \tag{125}$$

where $M_o = 4\pi R_o^3/3$ is the mass of the visible universe at the present time. This is an explicit prediction with no adjustable parameters, so should be possible to check with data. The small r/R_o expansion is readily obtained as:

$$\frac{\tilde{M}_g(r)\ \tilde{N}(r)}{M_o} \approx \left(\frac{r}{R_o}\right)^3 + \frac{3}{2}\left(\frac{r}{R_o}\right)^4 + \frac{9}{5}\left(\frac{r}{R_o}\right)^5 + O\left(\frac{r}{R_o}\right)^6) \tag{126}$$

This cubic dependence for small r means that the accumulated mass of the clusters increases linearly with the volume for small radius, consistent with the assumed actual *and local* homogeneity of the universe, the latter following from the quadratic dependence of the metric tensor in physical space on r.

It is more convenient to transform this from a dependence on r/R_o into z which can be directly measured. Since $r = r(z)$ from equation 122, we can define $M_g(z) = \tilde{M}_g(r(z))$ and $N(r) = \tilde{N}(r(z)$. Then we can substitute equation 122 into equation 125 to obtain $M_g(z)N(z)$ (after some manipulation) as:

$$\frac{M_g(z)\ N(z)}{M_o} = 3\left[\frac{2z+z^2}{1+z}\ -\ 2\ln(1+z)\right] \tag{127}$$

This blows up linearly asymptotically as z increases to infinity and $r \to R_o$, exactly as we found above.

The small z expansion is given by:

$$\frac{M_g(z)\ N(z)}{M_o} \approx z^3 - \frac{3}{2}z^4 + \frac{9}{5}z^5 + O(z^6) \tag{128}$$

As noted above, this cubic dependence on z could be quite useful in interpreting astronomical results.

It is important fo note how different the limiting values are for $M_g(z)N(z)$ and the volume, $V(z)$. The leading terms in the Taylor expansions of both vary as z^3 for $z << 1$, so $M_gN/V \to$ *constant* for very small values of z. This makes quantities per unit volume quite useful for small z. But for large

z things are more complicated, since $M_g N$ varies as z for large values, while V_z goes to a constant.

Note that if either $M_g(z)$ or $N(z)$ are known, either equation 127 or 128 can be used to find the other, assuming of course that M_o is known. We will suggest an additional hypothesis in Section 10.5 below to make this separation possible.

10.4 Mass per unit volume, $M_g(z)N(z)/V(z)$

The mass per unit volume as a function of z follows immediately from dividing the mass at z, the $M_g N$ of equation 127, by the volume at radius z, $V(z)$ given by equation 123. Since $M_o = \rho_o V_o$, the result is:

$$\frac{M_g(z)N(z)}{V(z)} \;=\; 3\,\rho_o\,\left[\frac{1+z}{z}\right]^3\left[\frac{2z+z^2}{1+z}\;-\;2\ln(1+z)\right] \qquad (129)$$

This increases linearly as $z \to \infty$. The small z expansion also begins linearly,

$$\frac{M_g(z)N(z)}{V(z)} = \;\rho_o\left[1+\frac{9}{4}z+\frac{27}{20}z^2+\cdots\right] \qquad (130)$$

Note the only parameter is ρ_o, the present density of the universe.

This can be contrasted with the same expression from the standard theory, given for example by equation 3 of Poggianti et al. [3] as:

$$\rho_m = \rho_c[\Omega_\lambda + \Omega_o(1+z)^3] \qquad (131)$$

These clearly have very different behavior for large values of z, our theory growing linearly in the limit as $z \to \infty$, the standard theory as z^3.

Note that to this point we have made no assumptions beyond our original hypothesis about the metric. In the next section we will make an additional hypothesis about N so we can separate mass and cluster number. So the results from this point on in this section will be dependent upon this new hypothesis as well.

10.5 Cluster number, $N(z)$, as a function of z

This section proposes to deduce $N(z)$ by itself by introducing the *additional* hypothesis that the average number of galaxies per unit volume in $\tau, \vec{\eta}$-space is constant.

Given our hypothesis, that the number is constant in $\tau, \vec{\eta}$-space, we define $\hat{n}(t)$ to be the *number density* in physical space at any physical time, t, $\hat{n}(t)$ must be proportional to $1/\delta(t)^3$ since the physical volume is increasing as $\delta(t)^3$; i.e.,

$$\hat{n}(t)\ \delta(t)^3 = constant = A \tag{132}$$

It follows that:

$$\frac{\hat{n}(t)}{\hat{n}(t_o)} = \left[\frac{t_0}{t}\right]^3 \tag{133}$$

since $\delta(t) = c\ t$.

But we also know that t, r and z are related, so we can rewrite equation 133 as a function of r as:

$$\frac{\tilde{n}(r)}{\tilde{n}(t_o)} = \left[\frac{R_0}{R_o - r}\right]^3 \tag{134}$$

So looking away in r (or back in time), the cumulative number of galaxies per unit volume, say $\tilde{N}(r)$ should be:

$$\tilde{N}(r) = 4\pi\ n(t_o) \int_0^r \left[\frac{R_o}{(R_o - r')}\right]^3 r'^2 dr' \tag{135}$$

It is easy to see by expanding this integral for small r (i.e. $r << R_o$), that $\tilde{n}(r) \approx n(t_o)$. And this is what we would have hoped for had we defined things correctly.

The exact solution for $r < R_o$ is given by:

$$\begin{aligned} \frac{\tilde{N}(r)}{n(t_o)} &= 4\pi \int_0^r \left[\frac{R_o}{(R_o - r')}\right]^3 r'^2 dr' \qquad (136) \\ &= 4\pi\ R_o^3 \left\{\frac{3}{2} + \frac{1}{2}\left(1 - \frac{r}{R_o}\right)^2 - 2\ \left(1 - \frac{r}{R_o}\right)^{-1} - \ln\left(1 - \frac{r}{R_o}\right)\right\} \end{aligned}$$

Or, defining $V_o = 4\pi R_o^3/3$:

$$\frac{\tilde{N}(r)}{n(t_o)\ V_o} = \frac{9}{2} + \frac{3}{2}(1 - r/R_o)^2 - 6\left(1 - \frac{r}{R_o}\right)^{-1} - 3\ln(\left(1 - \frac{r}{R_o}\right) \tag{137}$$

Note that $n(t_o)V_o$ is the number of clusters in the visible universe at the present time, obviously a very large number. We of course can only see them in their past. But we can see how the number appears to vary with distance away from us. So we should be able to make an estimate of the present cluster density, $n(t_o)$. The Taylor expansion about $r/R_o = 0$ is given by:

$$\frac{\tilde{N}(r)}{n(t_o)\ V_o} \approx \left(\frac{r}{R_o}\right)^3 + \frac{9}{4}\left(\frac{r}{R_o}\right)^4 + \frac{18}{5}\left(\frac{r}{R_o}\right)^5 + (5)\left(\frac{r}{R_o}\right)^6 + \frac{47}{7}\left(\frac{r}{R_o}\right)^7 + \cdots \tag{138}$$

We can define $N(z) = \tilde{N}(r(z))$ where $r/R_o = z/(1+z)$ and $1 - r/R_o = 1/(1+z)$. Substitution into equation 137 yields:

$$\begin{aligned}\frac{N(z)}{n(t_o)\ V_o} &= \left\{\frac{9}{2} + \frac{3}{2}(1+z)^2 - 6(1+z) + 3\ln(1+z)\right\} \\ &= -\frac{3}{2} - 6\ z + \frac{3}{2}(1+z)^2 + 3\ \ln(1+z) \\ &= -3z + \frac{3}{2}z^2 + 3\ln(1+z)\end{aligned} \tag{139}$$

Note that this increases *quadradically* in the limit as $z \to \infty$.

The small z Taylor expansion is given by:

$$\frac{N(z)}{n(t_o)\ V_o} \approx z^3 - \frac{3}{4}z^4 + \frac{3}{5}z^5 - \frac{1}{2}z^6 + \frac{3}{7}z^7 - \frac{3}{8}z^8 + \cdots \tag{140}$$

Clearly the cubic term dominates the approach to $z = 0$. Note that this leading cubic term is entirely a consequence of the Jacobian in the integral, and independent of the shape of the integrand. Nonetheless, we should (in principle at least) be able use it to determine $n(t_o)$ since we know V_o.

10.6 Cluster number per unit volume, $N(z)/V(z)$

We can divide equation 140 by equation 123 to obtain:

$$\frac{N(z)/V(z)}{n(t_o)} = \left[\frac{1+z}{z}\right]^3 \left[-3z + \frac{3}{2}z^2 + 3\ln(1+z)\right] \tag{141}$$

From the Taylor expansion of equation 140, we can see that the leading term of z^3 resolves the singularity at $z = 0$ leaving only:

$$\frac{N(z)/V(z)}{n(t_o)} = (1+z)^3[1 - 3\ z + 6\ z^2 - 10\ z^3 + 15\ z^4 + \cdots] \quad (142)$$

$$= 1 + \frac{9}{4}z + \frac{27}{20}z^2 + \frac{1}{20}z^3 - \frac{3}{140}z^4 + \frac{3}{280}z^5 + \cdots \quad (143)$$

10.7 Comparison of $N(z)/V(z)$ versus z for the data of [2] and [3]

In this subsection we will look at two sets of data: the low-z data of Abdullah et al. [2] which examines the GalWit19 data base, and the higher z data of Poggianti et al. [3]. The former is an extensive paper which attempts to count clusters and uses the data to evaluate the standard theory and simulations based on it. We consider here only their cluster data plotted as $N(z)/V(z)$ versus z. The Poggianti et al. paper is an older paper, one of several by the same authors, which examines clusters at two higher values of $z = 0.6$ and $z = 1.5$.

The Abdullah et al. data [2] are plotted alone in the top part of Figure 6. We have converted their numbers from $\mathrm{h}^3/\mathrm{Mpc}^3$ to $1/\mathrm{Mpc}^3$ using *their* value of $h = 0.678$. Since they did not include information below $z = 0.04$, we used an interative process to establish the value at the present time as $n(t_o) \approx 2250\ /\mathrm{Mpc}^3$. The theoretical curve fits the data well for $z \leq 0.12$, and over-shoots above this value for larger values of z. The three-term Taylor expansion of equation 143 fits equally well. This is consistent with the observation of Abdullah et al. who attributed the difference compared to simulations (above $z = 0.09$ in their case) to the incomplete data set for larger values. We agree. *Unlike their comparison to analysis, however, no part of our theory has to be attributed to dark energy or dark matter.*

Equation 141 together with the data Poggianti et al. [3] and the Abdullah et al. data are plotted in the bottom part Figure 6. The Poggianti et al. data were scaled (by them) by what they believed to be the value at $z = 0$. The numbers at the other two points were $N/V = 1.8$ for $z = 0.6$, and $N/V = 4.7$ for $z = 1.5$. Our theory for $N(0)V(0)/n(t_o)$ predicts $N/V/n(t_o) = 2.9$ for $z = 0.6$, and $N/V/n(t_o) = 7.5$ for $z = 1.5$. The ratios of the two data points to our theoretical values are 1.62 and 1.60 respectively, so the difference between ours and theirs is most likely due to the unknown coefficient $n(t_o)$. So we have multiplied their data by 1.6.

In summary it appears that our theory and especially the new hypothesis about the distributions of galaxies in $\tau - \vec{\eta}$ space looks promising. It should be noted that the same theory applies to properly performed simulations as well. Our theory suggests that $\delta(t_o) = c\ t_o$ is a length scale for the universe, not its extent which is presumed infinite. Experience in numerical simulations of homogeneous turbulence suggests strongly that the computational domain needs to be much larger than the characteristic length scale to minimize the effect of boundaries, typically by a factor of 10. We look forward to seeing the results of this comparison of theory and data by real astronomers.

10.8 Can we evaluate $M_g(z)$, the average cluster mass?

Given our solution for $N(z)$ above, the answer is yes – at least in principle. Dividing equation 127 by equation 139 yields:

$$\frac{M_g(z)\ n(t_o)}{\rho_o} = \frac{\left[\frac{2z+z^2}{1+z} - 2\ln(1+z)\right]}{-z + \frac{1}{2}z^2 + \ln(1+z)} \tag{144}$$

since the factor 3 multiplying both cancels and $M_o = \rho_o V_o$. Note that in the limit as $z \to \infty$, $M_g(z) n(t_o)/\rho_o \to 2/z$.

Alternatively we could write this as a ratio of their Taylor expansions:

$$\begin{aligned} \frac{M_g(z)\ n(t_o)}{\rho_o} &= \frac{z^3 - \frac{3}{2}z^4 + \frac{9}{5}z^5 - 2z^6 + \frac{15}{7}z^7 + \cdots}{z^3 - \frac{3}{4}z^4 + \frac{3}{5}z^5 - \frac{1}{2}z^6 + \frac{3}{7}z^7 + \cdots} \\ &= \frac{1 - \frac{3}{2}z + \frac{9}{5}z^2 - 2z^3 + \frac{15}{7}z^4 + \cdots}{1 - \frac{3}{4}z + \frac{3}{5}z^2 - \frac{1}{2}z^3 + \frac{3}{7}z^4 + \cdots} \\ &\approx 1 - \frac{3}{4}z + \frac{3}{80}z^2 - \frac{183}{320}z^3 + \frac{23649}{44800}z^4 + \cdots \end{aligned} \tag{145}$$

11 Logarithmic time

The comment of Dirac [1] at the beginning clearly foreshadows our work. In this section we review some aspects of our theory, some of which have also been anticipated.

Log-time	Seconds after the Big Bang	Period
-45 to -40	10^{-45} to 10^{-40}	Plank Epoch
-40 to -35	10^{-40} to 10^{-35}	
-35 to -30	10^{-35} to 10^{-30}	Epoch of the Grand
-30 to -25	10^{-30} to 10^{-25}	Unification
-25 to -20	10^{-25} 10 10^{-20}	
-20 to -15	10^{-20} to 10^{-15}	
-15 to -10	10^{-15} to 10^{-10}	Electroweak Epoch
-10 to -5	10^{-10} 10 10^{-5}	
-5 to 0	10^{-5} to 10^{-0}	Hadron Epoch
0 to +5	10^{0} to 10^{5}	Lepton Epoch
+5 to + 10	10^{5} to 10^{10}	Epoch of Nucleosynthesis
+10 to +15	10^{10} to 10^{15}	Epoch of Galaxies
+15 to +20	10^{15} to 10^{20}	

Table 1: Each row is defined in seconds after the Big Bang epochs of logarithmic time in cosmology with earliest at the top. The present time is approximately 4.9×10^{17} seconds (15.4 billion years) after the Big Bang.

11.1 Epochs of our evolution

From 23 we know that τ is logrithmically related to our physical time t. The description of a universe evolving in '*Epochs*' of logarithmic time is familiar to every cosmologist (see Table 1 taken from Wikipedia [47]).

While different physics dominates each region, our theory is probably the same for all after the Plank era, since the underlying assumptions would still be valid. The statistics would be the same, only the underlying mechanisms different.

11.2 Time measured in logarithmic increments

Aside from the comment by Dirac [1] near the end of his life and the musing of George [37] (from which this section was adapted), there appears to be no evidence (at least available to us) that it has ever been previously considered that different times should be applied to the Einstein field equations than to quantum mechanics. But even if we had thought of it, would we have noticed any difference?

Any cosmic (or logarithmic) time difference between two τ-times, say τ and $\tau + \delta\tau$, can be expressed in linear time increments by the difference of their Taylor series expansions; i.e.,

$$\begin{aligned} \delta\tau &= \ln[(t+\delta t)/t] - \ln[t/t] \\ &= \ln[(t/t)(1+\delta t/t)] - \ln(t/t) \\ &= \ln(t/t) + \ln(1+\delta t/t) - \ln(t/t) \\ &\approx (\delta t/t) + (\delta t/t)^2 + \cdots \end{aligned} \quad (146)$$

where t and $t + \delta t$ are the corresponding absolute times.

George [37] (using previously believed values for the age of the universe)) noted that:

> "There have been approximately 13.8 billion years ($t \approx 4.3 \times 10^{17}$ s) since the Big Bang. But mankind has only been on the earth for approximately 250,000 years. So even if we had been keeping careful track since then the differences we would have noticed between the hypothesized QSM time and linear time would have been $\delta t/t \approx 2.5{\times}10^5/13.7{\times}10^{12} \approx 3.4{\times}10^{-8}$. And the differences we would have needed to observe to discover a discrepancy are the square of this, or of order 10^{-15}. But we have been doing mechanics for only the past 500 years, so even had we started measuring carefully at Galileo, $\delta t/t \approx 500/13.7 \times 10^{12} = 3.6 \times 10^{-11}$. So the leading error term would have been of order 10^{-21}, and clearly beyond our ability to distinguish from experimental data alone. Only by trying to make sense of things that happened billions of years ago would we have noticed our equations don't balance. But of course we have done that now."

Figure 8 makes clear WHY we would not have noticed a problem. By differentiating equation 23, it follows immediately that:

$$\frac{d\tau}{dt} = \frac{1}{t}. \quad (147)$$

We are now at 15.4 billion years on the plot. The slope $d\tau/dt$ has been very nearly constant over all of our human existence. And the same is true as well for most of our previously observable universe (i.e., before the Hubble telescope). Only now can we see far enough back in time to see any difference from a simple linear relationship, $t \propto \tau$. So only now do we see the inconsistencies in our previous theories.

11.3 The non-vanishing horizon

An interesting consequence of our theory is that the last star will *not* vanish over the horizon. The universe is expanding at precisely the right rate to prevent that from happening. This should be quite a relief for those who found the previous conclusions depressing.

But another question which is related is: Will the last star burn out? The answer is not obvious to us and we leave it to others to speculate. Surely since G and c are the properties of nature, there is no reason there will be less of them. But if quantum mechanics really functions on its own time scale (as suggested in the quotation from Dirac at the beginning of this paper), then the answer is probably yes. But if not.....? We leave it to the quantum field theorists and nuclear physicists to reason this out, since it is clearly beyond our capabilities – at least at the moment.

12 Summary and Conclusions

This paper explored the consequences of the simple hypothesis that we live in an infinite universe in which both time and space evolve with time when scaled by the same time-dependent length scale. This is fundamentally different from the traditional FLRW approach where only space expands with time. We have not replaced Einstein's Field equations, but we have instead found a different solution to them – one which evolves in time, but does so in a way so that the acceleration terms of the field equations vanish identically. A direct consequence is that the problematical '*critical density*' concept vanishes completely, and with it the need for either *Dark Energy* or *Dark Matter*.

We removed the time evolution by simply considering a universe in scaled coordinates, say τ and $\vec{\eta}$, to be a maximally symmetrical Minkowski/de Sitter zero curvature space of infinite extent that remains fundamentally unchanged during its evolution. Then we propose that this coordinate system is related to our physical coordinate system $(t, \vec{x})$ by stretching the coordinates with a time-dependent length scale $\delta(t)$. The statistics of this stretched universe are inhomogeneous in space, $\vec{x}$, and so vary in both space and time, t.

In effect, we have simply placed Einstein's original hypothesis of a static universe into a coordinate system in which both time and space expand together. The metric tensor is constant in $\tau, \vec{\eta}$-space. but in physical coordinates evolves with time. A consequence is that even though space and time are expanding, the zero values of the Riemann and Ricci tenors mean that Einstein's stress-energy tensor reduces to a single function multiplying the metric tensor. And that function depends only on the gravitational constant G, the speed of light c, and the time-dependent length scale δ.

The rate of expansion is the speed of light and we show that $\delta(t) = c\ t$ where t is the age of the universe. So both of these physical constants, G and c, are related directly to an initial condition - in this case the Big Bang or its residual when gravity becomes important. The second is undoubtedly related to why nothing can exceed the speed of light, since nothing can propagate faster than the underlying space in which it propagates. The analogies with wave propagation in hydraulics and compressible flow are intriguing, but we have not (yet at least) pursued them.

A direct consequence of the assumptions of a static universe in similarity coordinates is that the length scale is linearly dependent on physical time, t, and that $\tau = \ln t/t_1$ where t_1 is a virtual origin proportional to the Planck time and indicative of the Big Bang. Thus the proper time (and similarity independent variable) τ depends on the logarithm of t, a result consistent with the speculations of George [37, 38] who examined the consequences of a log-time assumption. Interestingly, Caballero [48] (see also [49]) has recently argued using the Wolfram Model of fundamental physics ([50]) that logarithmic time can be proved to be generically the same as the total information content of the Universe. This provides a striking parallel to the deduction herein that initial conditions establish directly the gravitational constant G, and that the expansion rate is proportional to the speed of light c, both fundamental constants of physics.

The linear relation between the length scale $\delta(t)$ and t implies that the Hubble parameter is inversely proportional to the age of the universe; i.e., $H = 1/t$. Our prediction that the Hubble parameter varies as $1/t$, where t is age of the universe would appear to contradict current wisdom. But Carroll [13] has noted this is the desired result for the very early universe as well. We show that $H = 1/t$ implies that $H = H_o \times [1 + z]$ where H_o is the present value. We show this equation to be in excellent agreement with the recent results of Yu et al. for $H_o = 63.6$ km/s/Mpc. This corresponds to a universe which is 15.4 billion years old. Our older universe explains a number of recent results including the age of the Methuselah star (14.5 billion years), and recent inferences from supernovae and gravitational lensing, all of which appear to be older than presently assumed values for the universe (13.8 billion years).

Our theory with no adjustable constants also examines how the redshift parameter, z affects previous analyses of supernovaes. We find excellent agreement with the 'uncorrected' results of Hamuy et al. [27], Reiss et al. [15], Perlmutter et al. [5] and Knop et al. [6]. Given the uncertainty in the absolute magnitude of each supernova, both the corrected and uncorrected data agree with our theory using slightly different values. But we argue using the scaled Maxwell's equations that the uncorrected data is preferred, since radiation spectra are shifted in both amplitude and wavenumber.

We also show that our theory explains the so-called '*Worst Prediction in the History of Physics*', the enormous discrepancy between the predictions QFT, of quantum field theory, and observations – 10^{120}! The fact that time is not measured in linear increments means that energy and mass are not conserved quantities, only G and c are. In fact both rest mass energy and mass densities decay as $e^{-2\tau}$, or equivalently $[c/\delta(t)]^2 = 1/t^2$. We are not the first to notice that changing time affects conservation of energy (c. f. [36, 13]), but perhaps we are in this context.

We show that our theoretical predictions for the growth of Planck scale disturbances near the beginning are consistent with the Planck observations of the scale of the inhomogenieties, the Cosmic Background Radiation. And by considering when the temperature was 3000 degrees K, we suggest that in fact, they date not to 380,000 years after the big bang, but to 14 million years instead.

And finally, we confess our limitations in considering the enormous quantity of astronomical data. But we try to leave a road map for astronomers to follow should our theory prove interesting.

In summary, we have proposed a new theory based on a single time-dependent length scale, $\delta(t)$. It turns out after extensive analysis using curvilinear coordinates that $\delta(t) = c\ t$. We argue that average energy and average mass are not conserved. In fact their densities vary inversely as $1/t^2$.

Our theory found a solution to Einstein's equations which evolves logarithmically with time. But the equations we used were exactly Einstein's equations. It was our solution that gave surprising answers. But was this a clue? Could it be that those underlying 'Laws of Nature' should have been expressed using logarithmic time as well. If so, 'mass' would not be the mass we thought it was, nor 'energy' the same energy. Like the '42' in the *Hitchhiker's Guide to the Galaxy*, we leave this question for the next generation of physicists to answer. Wouldn't it be ironic if given well-known Einstein's aversion to quantum mechanics, it is the application of his equations that proved quantum mechanics was right all along, only our 'known' laws were in error.

Acknowledgements

These ideas have evolved over the last few years, mostly in discussions between the authors, neither of whom were trained in astronomy or cosmology. Whether or not we have made a significant contribution, we are extremely grateful those experts who wrote books and made videos of their lectures available. It has been an exciting journey thanks to their efforts. Many former students and friends made helpful comments, but the encouragement and suggestions of Azur Hodzic (Danish Technical University) and Jose Manuel Rodriguez Caballero (University of Tartu, Estonia) were especially helpful. We would also like to acknowledge the gracious proprietor, Camilla Eriksson, and the friendly and comfortable environment of her cafe 'Kaffebubblan' in Mölndal, Sweden where many of our discussions took place. Finally we acknowledge the contribution of April Howard, wife of WKG, who listened patiently to many versions of this theory, mostly wrong. But in doing so helped to clarify the holes. Last but surely not least, we acknowledge the work and dedication of the many '*et al*'s. As mostly retired professors, we know well what effort hides behind that simple citation.

WKG and TGJ both retired from Chalmers Technical University in 2010, WKG as Professor of Turbulence and Director of the Turbulence Research Laboratory (www.turbulence-online.com), TGJ as Docent in the same laboratory. Both were instrumental in the Chalmers International Turbulence Masters program which moved to Ecole Centrale de Lille upon their retirement.

A Mathematical Details

A.1 Notation and Jacobians

We choose the notation of Grinfel [20] for its simplicity. We keep the speed of light explicit and do NOT absorb c into the time t. While analytically more complicated, it simplifies interpretation later. The coordinates with unprimed indices, z^μ, are used for $\tau, \vec{\eta}$-space, whereas the coordinates with primed indices, $z^{\mu'}$, are used for the physical coordinates, $t, \vec{x}$. The indices 0 (or $0'$) are used for τ and t respectively.

We define the primed coordinates to correspond to our physical space (ct, x, y, z) as follows:

$$z^{0'} = t \,,\; z^{1'} = x \,,\; z^{2'} = y \,,\; z^{3'} = z, \tag{148}$$

The un-primed coordinates, z^μ represent $\tau, \vec{\eta}$ and are related to the physical space coordinates as follows:

$$z^0 = c \int_0^{z^{0'}} \frac{d\zeta}{\delta(\zeta)} \,,\; z^1 = \frac{z^{1'}}{\delta(z^{0'})} \,,\; z^2 = \frac{z^{2'}}{\delta(z^{0'})} \,,\; z^3 = \frac{z^{3'}}{\delta(z^{0'})}, \tag{149}$$

The corresponding Jacobians, $J^\mu_{\mu'} = \partial z^\mu / \partial z^{\mu'}$ and $J^{\mu'}_\mu = \partial z^{\mu'} / \partial z^\mu$, take on the following form:

$$J^\mu_{\mu'} = (1/\delta) \begin{Bmatrix} c & 0 & 0 & 0 \\ -[\dot{\delta}/\delta] z^{1'} & 1 & 0 & 0 \\ -[\dot{\delta}/\delta] z^{2'} & 0 & 1 & 0 \\ -[\dot{\delta}/\delta] z^{3'} & 0 & 0 & 1 \end{Bmatrix} \tag{150}$$

$$J^{\mu'}_\mu = \delta \begin{Bmatrix} 1/c & 0 & 0 & 0 \\ [\dot{\delta}/\delta \; c] z^{1'} & 1 & 0 & 0 \\ [\dot{\delta}/\delta \; c] z^{2'} & 0 & 1 & 0 \\ [\dot{\delta}/\delta \; c] z^{3'} & 0 & 0 & 1 \end{Bmatrix}, \tag{151}$$

We note that $\delta^\mu_\nu = J^\mu_{\mu'} J^{\mu'}_\nu$.

A.2 Basis vectors

The covariant basis vectors in $\tau, \vec{\eta}$–space are by hypothesis:

$$\begin{Bmatrix} \vec{b}_0 & = & (1,0,0,0) \\ \vec{b}_1 & = & (0,1,0,0) \\ \vec{b}_2 & = & (0,0,1,0) \\ \vec{b}_3 & = & (0,0,0,1) \end{Bmatrix} \tag{152}$$

And in $t, \vec{x}$)-space (physical space) by:

$$\begin{Bmatrix} \vec{b}_{0'} & = & (1/\delta, -x\dot{\delta}/c\delta^2, -y\dot{\delta}/c\delta^2, -z\dot{\delta}/c\delta^2) \\ \vec{b}_{1'} & = & (0, 1/\delta, 0, 0) \\ \vec{b}_{2'} & = & (0, 0, 1/\delta, 0) \\ \vec{b}_{3'} & = & (0, 0, 0, 1/\delta) \end{Bmatrix} \tag{153}$$

where $\dot{\delta} = d\delta(t)/dt$. Note that the basis vectors make it clear that our assumed physical space is not Cartesian.

The covariant and contravariant metric tensors in physical space, say $g_{\mu'\nu'}$ and $g^{\mu'\nu'}$, are readily computed to be those given by equations 10 and 11. In $\tau, \vec{\eta}$-space the Christoffel symbols are zero. But ten of the Christoffel symbols of the second kind in physical space are non-zero. They are:

$$\Gamma^{0'}_{0'0'} = \Gamma^{1'}_{0'1'} = \Gamma^{1'}_{1'0'} = \Gamma^{2'}_{0'2'} = \Gamma^{2'}_{2'0'} = \Gamma^{3'}_{0'3'} = \Gamma^{3'}_{3'0'} = -\left[\frac{\dot{\delta}}{\delta}\right] \tag{154}$$

$$\Gamma^{1'}_{0'0'} = -x\left[\frac{\delta\ddot{\delta} - \dot{\delta}^2}{\delta^2}\right] \tag{155}$$

$$\Gamma^{2'}_{0'0'} = -y\left[\frac{\delta\ddot{\delta} - \dot{\delta}^2}{\delta^2}\right] \tag{156}$$

$$\Gamma^{3'}_{0'0'} = -z\left[\frac{\delta\ddot{\delta} - \dot{\delta}^2}{\delta^2}\right] \tag{157}$$

$$\tag{158}$$

Clearly only the last three involve space. The Christoffel symbols of the first-kind can be obtained by lowering the upper index using the metric tensor.

B Postscript

Between us (WKG and TGJ) we have over 100 years experience as scientific researchers in fundamental mechanics and applied physics —- not one second of it in general relativity, cosmology, or astronomy. So everything we have learned, we have learned in retirement, mostly from the internet and during the Covid-19 pandemic. We have both moved into new fields before, always by first reading books and journals, attending lectures or special courses, and most importantly by attending professional meetings so we could confer with experts. None of that was possible the past few years. So if at times we sound naïve, we probably are. And we apologize if we have neglected or misunderstood things that are obvious to those working in the field, or if we have had a limited number of citations of recent work. Not much else has been available to us, nor could we judge its quality if it had been. So we have had to depend heavily on a few sources, hopefully good ones.

WKG's interest in this subject was tweaked by a Canadian radio program "As It Happens", heard on Boston Public Radio (WGBH) while driving late at night to attend the American Physical Society/Division of Fluid Dynamics meeting in Boston in November 2015. The radio host was interviewing three scientists about dark matter and dark energy, both subjects of which while interesting had always been nothing more than a curiosity. But never before had it been clear that what was being discussed was really mostly a failure of classical mechanics to describe the observations. During lunch at the meeting the next day with two former students (Clay Byers and Marcus Hultmark, both now professors) while discussing homogeneous turbulence and its time and length scale evolution, the idea that the missing mass and energy might be related to time was born. This ultimately resulted in a paper and several presentations [37, 38]. But it quickly became clear that much more sophisticated mathematical tools were needed to advance beyond mere speculation.

WKG's return to Sweden in 2018 provided the perfect opportunity to link up with a former colleague from Chalmers, TGJ, who had retired about the same time. So together we began to meet regularly to teach ourselves about astronomy, curvilinear coordinates and general relativity — sharing notes, ideas and many misunderstandings. The youtube.com online-courses of L. Susskin, (Stanford), A. Maloney (McGill), A. Guth (MIT) and P. Grinfeld (Drexel) were especially helpful. But there were many others as well. Their efforts and generosity in sharing online made our effort possible.

This paper is an outgrowth of that effort. History alone will judge whether we have made an important contribution, or any contribution at all. At our age as septuagenarians, each contribution might well be our last. So most important to us is not another published paper or battle won over hostile reviewers (of which we have had many), but whether we have stimulated others to think about this problem (or others) in new ways. Regardless, to paraphrase the inspirational Dr. Becky (of Oxford University and youtube.com fame): We really have enjoyed having a 'ring-side-seat' at the best time in the history of the world for studying (and learning about) astronomy and cosmology.

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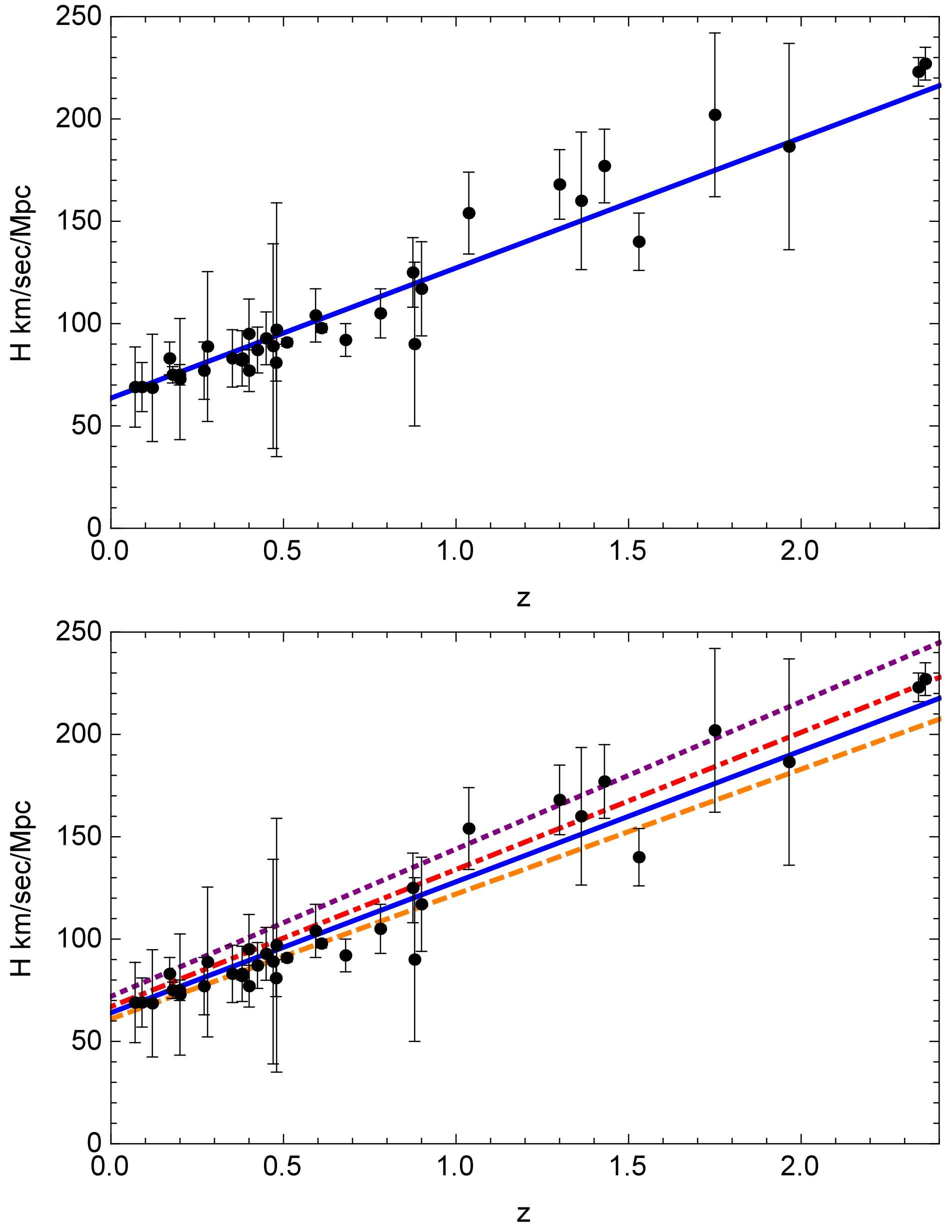

Figure 1: $\tilde{H}(z)$ versus z for data of Yu et al. 2018 [4]. Top: Equation 76, $\tilde{H}(z) = H(t_o)\ [1 + z]$, with best fit $H_o = H(t_o) = 63.6$ [km/s/Mpc] (solid line). Bottom: Same plot but with alternative values $H(t_o) = 61$ (dashed line), 67 (dash-dotted line) and 72 (dotted line).

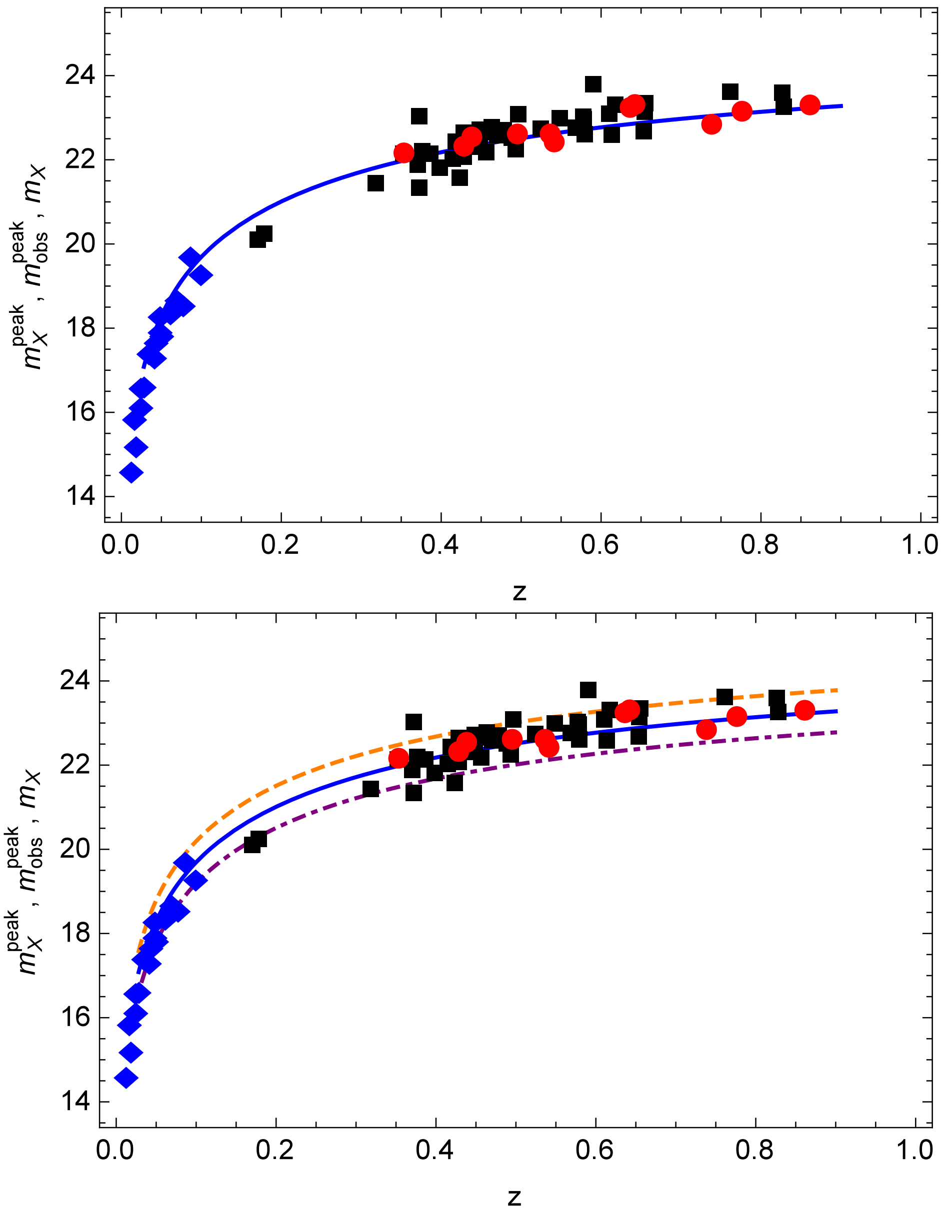

Figure 2: Data from column 4 of both Table 1 and Table 2 in Perlmutter et al. [5] and data from Table 3, column 2 of Knop et al. [6] are plotted versus z. The ordinate labels correspond to the table column headings. Top: Best fit of equation 91, $m = 5\ log_{10}[z/(1+z)] + 43.4 + M$, using $M = -18.5$. Bottom: Three theoretical curves using equation 91 have also been plotted using values for $M = -18.0, -18.5$ and -19.0 respectively.

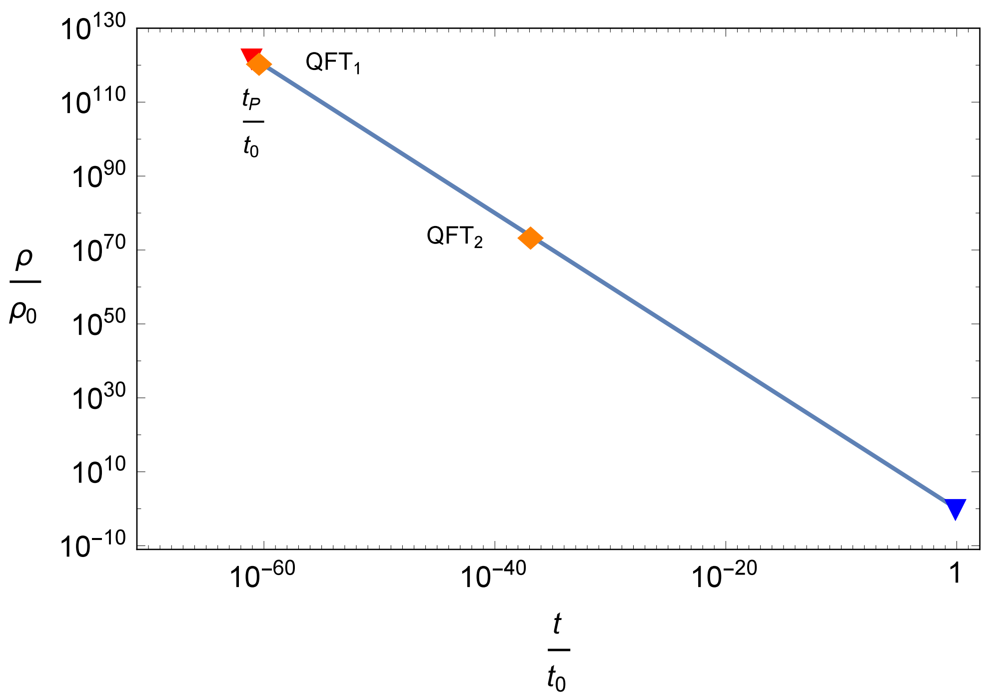

Figure 3: Plot of equation 100 showing 122 decades of mass density normalized by the present value versus time normalized by the age of the universe. The blue triangle is the present value. Also shown are the QFT1 value and the QFT2 value (orange diamonds), both normalized by the present day density of Abdullah et al. [2].

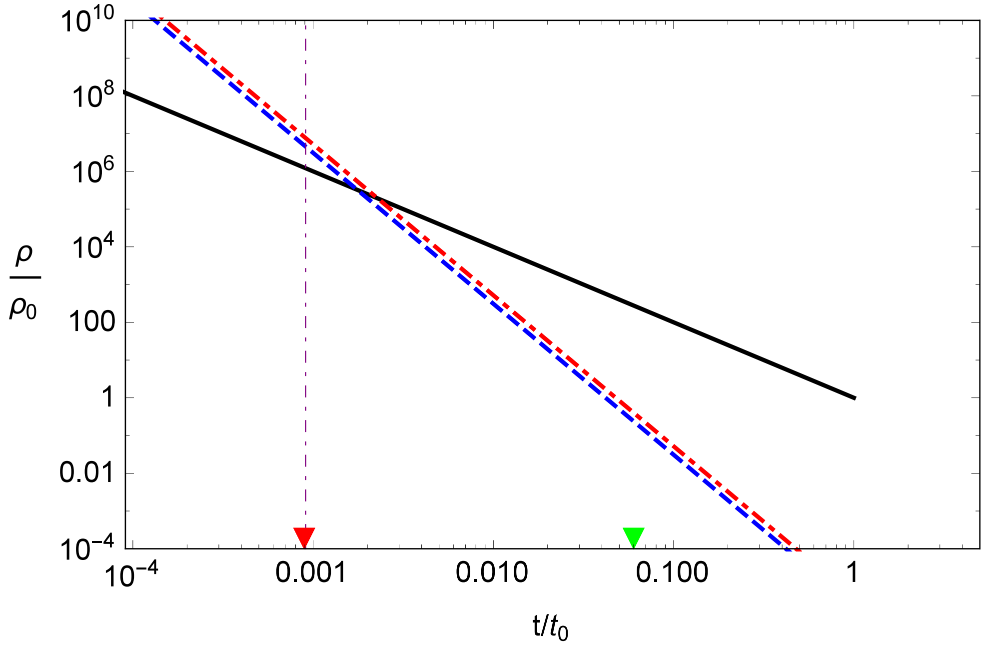

Figure 4: Blow-up of Figure 3 showing only times after $t/t_o = 10^{-4}$. The black line is equation 100, and the dashed lines are the radiation estimates of equation 109 and 110. For reference purposes, we have also shown on the plot the time associated with the Cosmic Background Radiation (red triangle) when the temperature was 3000 degrees K corresponding to $z = 1100$. The green triangle indicates the age of the Methuselah star (14.5 billion years).

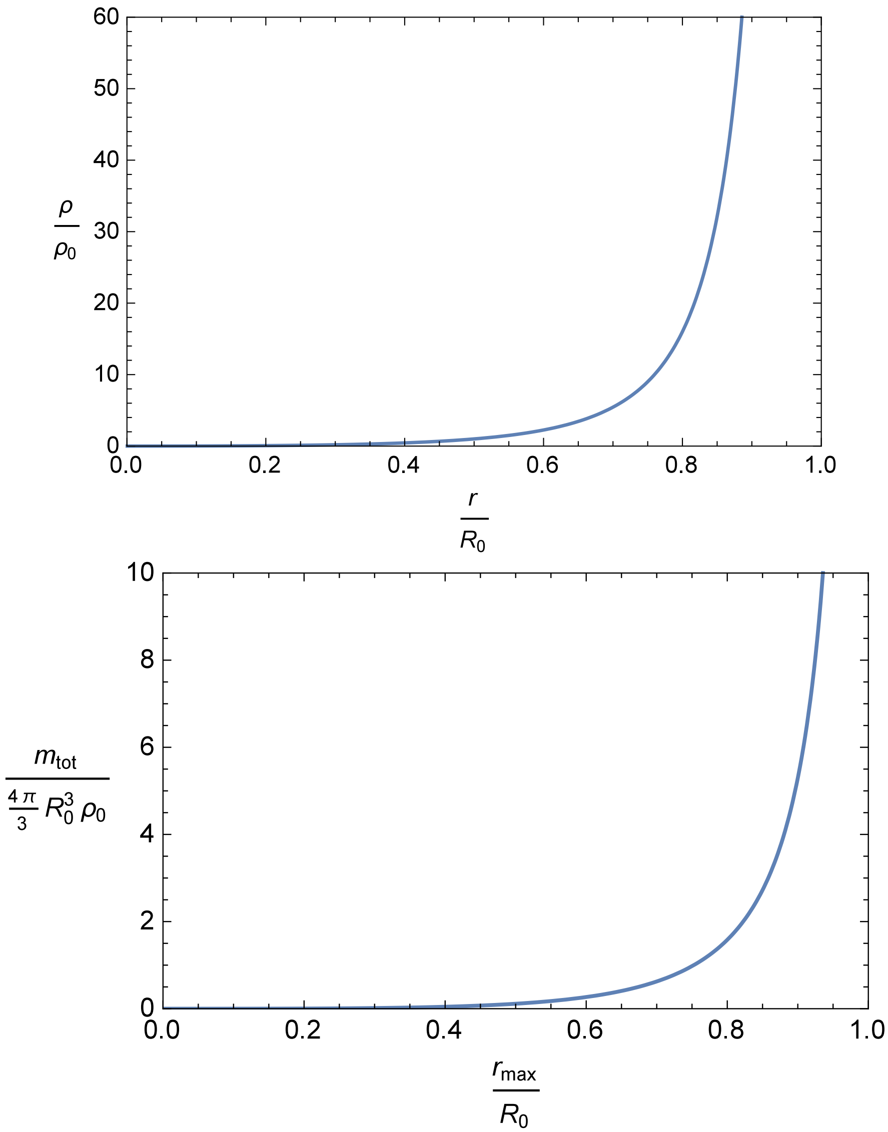

Figure 5: Upper: Plot of equation 119 looking back in time and showing how density varies with distance from an observer, equation 119. Note the apparent singularity as r approaches the radius of the visible universe, $R_o = c\ t_o$. Lower: Plot of equation 120 looking back in time and showing how the cumulative mass varies with distance from an observer over which the integral is computed, r_{max}. The cumulative mass is normalized by the mass at present, $4\pi R_o^3 \rho_o$.

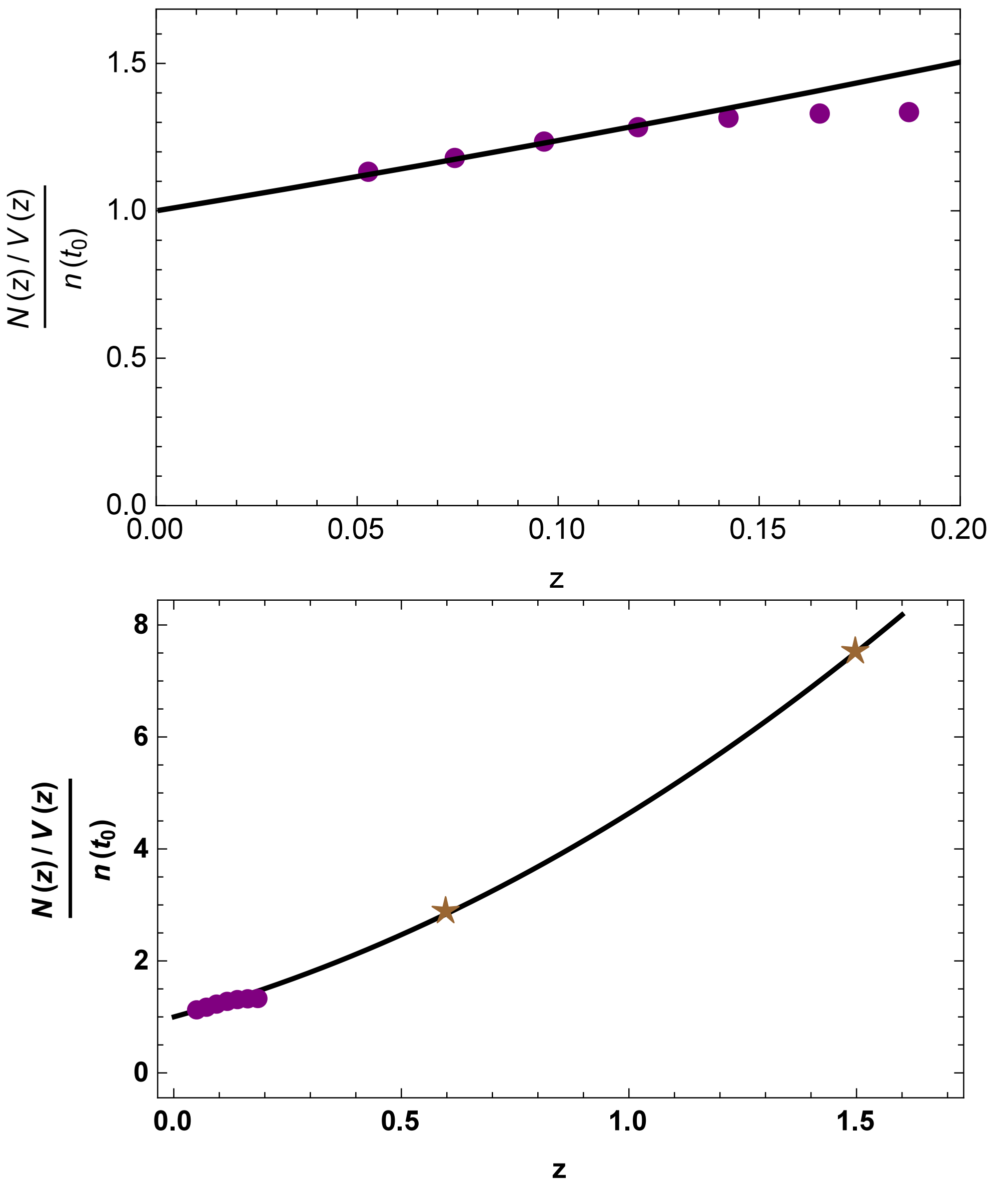

Figure 6: Upper:The cumulative cluster number data of Abdullah et al.[2] plotted with equation 141 using $n(t_o) = 2250$. Lower: Linear-linear plot of equation 141 and the cumulative cluster number data of Poggianti et al. [3] and Abdullah et al. [2]. For the latter we used $n(t_o) = 2250$.

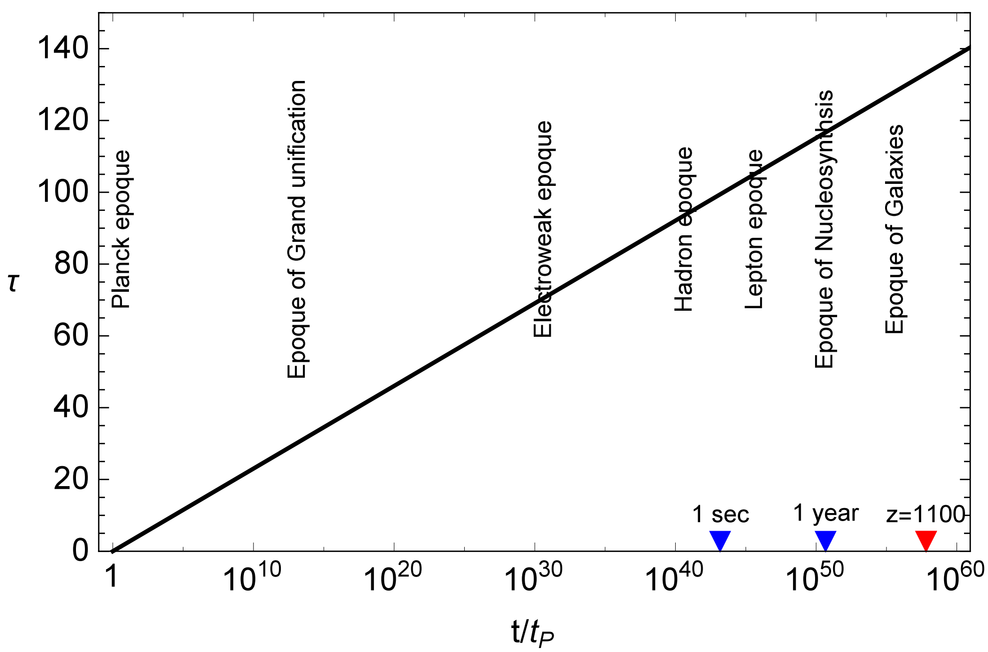

Figure 7: Figure showing how the times, $t/t_P = (t_1/t_P)\ e^{\tau}$ and $\tau = \ln\ [(t/t_1)(t_1/t_P)]$ are related, where t_P is the Planck time and t_1 is the virtual origin. For plotting purposes we have chosen $t_1/t_P = 1$. The 'Epochs' from Table 1 have been identified, along with 1 second, 1 year, and the time for which $z = 1100$.

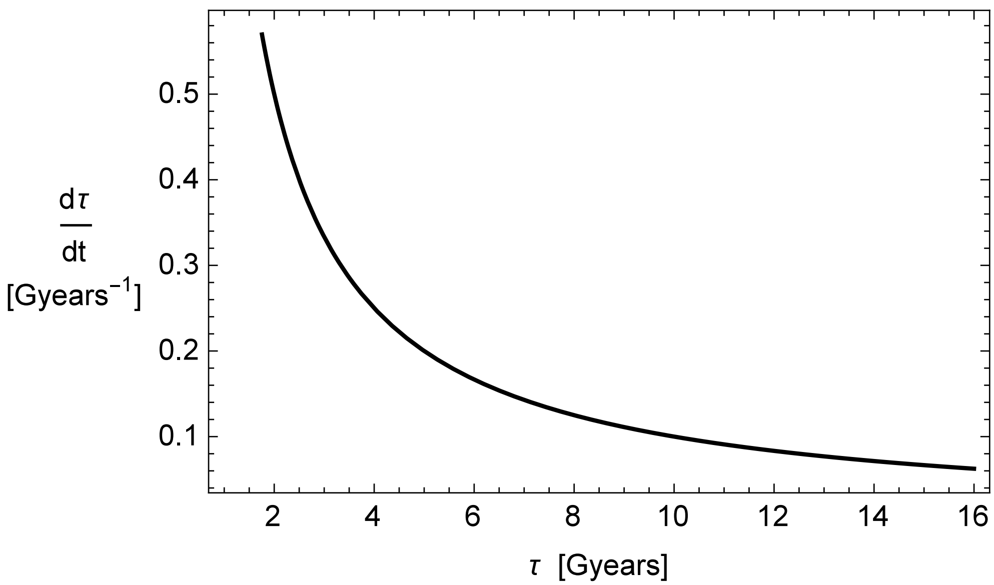

Figure 8: Figure plots equation 147 showing how $d\tau/dt$ varies with time t. The green triangle shows for reference the birth time of the Methuselah star, and the magenta triangle the birth time of the earth. Our human existence could be represented on this graph by a very thin vertical line at about 15.4 billion years. For the entire time of human existence $d\tau/dt \approx const$, so any difference between t and τ would have been undetectable. Only by looking back over billions of years could any deviation from linearity have been noticed.