Poster Session K01.3 APS DFD 2026 Orlando, Fl (download paper and poster from www.turbulence-online.com)

How astronomers misinterpret their data to erroneously obtain 'dark matter'

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$$-\frac{V_c^2}{r} = -\frac{\partial \Phi}{\partial r}$$

Also it is common to define a '*circular* velocity', say V_c by using all the terms on the left-hand side except the time derivative; i.e.,

$$-V_c^2 = -V_\phi^2 + \left\{ \frac{\partial (r V_r^2)}{\partial r} + r V_r^2 \frac{\partial \ln \rho}{\partial r} + \frac{\partial (V_\phi V_r)}{\partial \phi} + [(V_\phi V_r)] \frac{\partial \ln \rho}{\partial \phi} + r V_r V_z \frac{\partial \ln \rho}{\partial z} - r \frac{\partial}{\partial r}\left(V_z^2\right) - r V_z^2 \frac{\partial \ln \rho}{\partial r} - r \frac{\partial \overline{V_z^2}}{\partial r} \right\} \qquad (20)$$

Except near the core region, the first term, V_ϕ^2 dominates.

Since the velocities are small compared to the speed of light, Gauss' gravitational law applies. In cylindrical polar coordinates it is given by:

$$\left[\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \right] \Phi(r,\phi,z) = 4\pi G\, \rho(r,\phi,z), \qquad (24)$$

where G is the universal gravitational constant.

'Dark Matter' in exponentially decaying density galaxies

$$\overline{\rho_{eg}}(r,\phi,z) = \rho_o \left[exp(-z^2/2\sigma_z^2) \right] exp(-r/z_r)$$

$$\overline{V^2_{\phi g}}(r,0) = G M_{gc} \frac{r^2}{2z_r^3} \left[I_0\left(\frac{r}{2z_r}\right) K_0\left(\frac{r}{2z_r}\right) - I_1\left(\frac{r}{2z_r}\right) K_1\left(\frac{r}{2z_r}\right) \right], \qquad (5$$

$$\langle V^2_{\alpha g}(r) \rangle = r \frac{\partial \langle \Phi_{gc}(r) \rangle}{\partial r} = r \int_{-\infty}^{\infty} dz \frac{\partial \overline{\Phi}_{gc}(r,z)}{\partial r} = 2GM_{gc}\left[1-(1+(r/z_r))\exp(-r/z_r)\right] \qquad (53)$$

$$4\pi G \langle \rho_{dm}(r) \rangle = \frac{1}{r}\frac{\partial}{\partial r}\left\{ \frac{1}{z_m(r)} \left(r \frac{\partial \Phi_{gc}(r)}{\partial r} \right) - r \frac{\partial \overline{\Phi}_{gc}(r,0)}{\partial r} \right\} \to \left[\frac{2M_{gc}G}{z_r^3} \right] \left\{ \left(\frac{z_r}{z_m} \right) \exp(-r/z_r) + \frac{1}{2(r/z_r)^3} \right\}$$

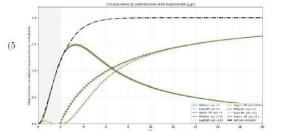


Figure 10: Generic plots of $\langle V^2_{gc}/GM_{gc} \rangle$ and (z_m/z_r) times $\overline{V^2_{gc}}(r,0)/GM_{gc}$ and $\Delta V^2_{dmgc}(r)/M_{gc}$ for Gaussian-exponential density. This parallels the way astronomical data would be fitted to data. The values of z_t/z_r are 0, 0.05 and 0.1. $z_m(r)/z_m(\infty) = 1 - exp(-Ar/z_r)$. $z_m(\infty)/z_r$ were calculated from equation 62. Also shown are the two '*dark matter*' contributions (equations 91 and 95) which would have been inferred if the integration over z had not been included in the analysis.

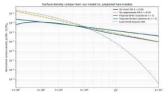


Figure 11: Plots showing the dark matter density profile, $\langle \rho_{dmgc}(r) \rangle$ (equations 92 and 98) that would have been imagined to be true if analyzed by the centerline equations instead of the vertically integrated ones. Also shown are two commonly used empirical 'dark matter halo' profiles of Burkert [8] and NFW [68] which nearly parallel our approximate '*fake dark matter*' profile.

'Dark Matter' in Power Law Density Galaxies

$$\overline{\rho}_{pl}(r,z)\rangle = \rho_o \left\{ 1 + (z/z_t)^2 + (r/z_r)^2 \right\}^{-5/2}$$

$$\overline{V^2_{pl}}(r,0) = \frac{r\,\partial \overline{\Phi}_{pl}(r,0)}{\partial r} = \frac{3GM_{pl}}{2} r^2 \int_0^\infty du \frac{\left[1 + r^2/(z_r^2+u)\right]}{(z_r^2+u)\sqrt{z_t^2+u}}$$

$$\langle V^2_{\alpha pl}(r) \rangle = \langle \frac{r\,\partial \Phi_{pl}(r,z)}{\partial r} \rangle = \frac{8\pi G \rho_o z_r^2 z_t}{3} \frac{r^2/z_r^2}{[1+(r/z_r)^2]}$$

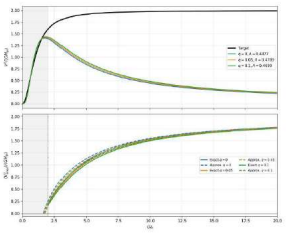


Figure 12: Upper: Generic plots of $\langle V^2_{pl}/GM_{pl} \rangle$ and (z_m/z_r) times $\overline{V^2_{pl}}(r,0)/GM_{pl}$ and $\Delta V^2_{dmpl}(r)/M_{pl}$ for power law density. The values of z_t/z_r are 0, 0.05 and 0.1. $z_m(r)/z_m(\infty) = 1 - exp(-Ar/z_r)$. $z_m(\infty)/z_r$ calculated from equation 79. This parallels the way data are usually treated. Lower: The '*dark matter*' contributions from equations 97 and 99 that would have been inferred had the theory not been integrated over z.

A home-recipe for creating your own 'dark matter' in spiral galaxies

- Find measurements obtained with great difficulty and expense of the velocities and densities of the stars in your favorite spiral galaxy.
- Make ***'local 'averages'*** over the plane of the galaxy (r and ϕ) to enable plots of velocity and density versus radius r. Or even better, find someone else's results.
- **Ignore the fact that a spiral galaxy is not an infinitesimally thin disk.** It has a thickness and a z-profile no matter how thin, and all measurements are actually an integral through the galaxy – not just 'surface' averages.
- **Analyze with the wrong equations.** If the measurements are integrated over z, then the equations predicting them must be as well.
- Correct your incorrect calculation by adding '**dark matter**'.

What did they really measure?

$$\overline{\overline{V^2_{\alpha\, meas}}}(r;\Delta z(r)) = \overline{\left[\frac{1}{\overline{\overline{N_{col}}}(r)}\right]} \overline{\left[\overline{N_{col}}(r)\right]} \left\{ \frac{1}{\Delta z} \int_{-\Delta(r)/2}^{\Delta z(r)/2} dz' \, \overline{\overline{V^2_{\alpha *}}}(r,z') \right\} \qquad (128)$$

Clearly the term in curly brackets, $\overline{\overline{V^2_\alpha}}(r,z)$, is just the star velocities averaged azimuthally and integrated over the interval from $z = -\Delta z/2$ to $z = +\Delta z/2$. If $\Delta z > z_t$ by a lot, the integral is just the definition of the spatially averaged velocity, $\langle V^2_\alpha(r) \rangle$. This is our model. On the other hand, if $\Delta z << z_t$, it can approximated as Δz times its value at the centerline ($z = 0$); i.e., $\overline{\overline{V^2_{\alpha *}}}(r,0)$. This has been the basis of all previous analyses.

So which is right? Approximating $V_c(r,z)$ using the centerline (z = 0), or treating it as a spatially averaged quantity including z?

- Actually both are right – depending on how thick the galaxy is relative to the resolution in z.

Since we assume $\Delta z << z_t$ we can approximate the integral by the first two terms in its Taylor expansion in z; i.e.,

$$\int_{-\Delta(r)/2}^{\Delta z(r)/2} dz' \, \overline{\overline{V^2_\alpha}}(r,z') \approx \Delta z \, \overline{\overline{V^2_\alpha}}(r,0) + \frac{(\Delta z)^3}{12} \overline{\frac{\partial^2 \overline{V^2_\alpha}(r,0)}{\partial z^2}} + \cdots \qquad (137)$$

where we have assumed the velocity profile to be symmetric in z and neglected the first derivative term.

So in this limit the measured velocity is exactly what seems to have been assumed by the astronomical community for years; i.e.,

$$\overline{V^2_{\alpha\, meas}}(r;\Delta z(r)) = \overline{\left[\frac{1}{\overline{N_{col}}(r)}\right]} \overline{\left[N_{col}(r)\right]} \left\{ \overline{V^2_\alpha}(r,0) + \frac{(\Delta z)^2}{12} \overline{\frac{\partial^2 V^2_\alpha(r,0)}{\partial z^2}} \right\} \qquad (138)$$

For the $\Delta z >> z_t$, the integral over z is effectively infinite. So the measured velocity is:

$$\overline{V^2_{\alpha\, meas}}(r;\Delta z(r)) \approx \overline{\left[\frac{1}{\overline{N_{col}}(r)}\right]} \overline{\left[N_{col}(r)\right]} \left\{ \frac{1}{\Delta z} \int_{-\infty}^{\infty} dz' \, \overline{V^2_\alpha}(r,z') \right\} \qquad (129)$$

$$\approx \overline{\left[\frac{1}{\overline{N_{col}}(r)}\right]} \overline{\left[N_{col}(r)\right]} \frac{\langle V^2_\alpha(r) \rangle}{\Delta z}$$

Clearly in this limit, the measured velocities are exactly equal to the spatially-averaged velocity, $\langle V^2_\alpha(r) \rangle$.

The approximation on the left is justified by arguing that the galaxy profile is very thin. But actually that justifies the assumption on the right, since it would require infinitesimal resolution to resolve the peak. Also, only the left requires dark matter to fill in the gap.

An expanded presention about this poster.

- The slides below provide a bit more motivation and explanation for the poster.
- It and various drafts of this work and other posters and presentations are available on our website at www.turbulence-online.com

A home-recipe for creating your own ‘dark matter’ in spiral galaxies

- Find measurements obtained with great difficulty and expense of the velocities and densities of the stars in your favorite spiral galaxy.
- Make ***‘local ‘averages’*** over the plane of the galaxy (r and ϕ) to enable plots of velocity and density versus radius r. Or even better, find someone else’s results. Be careful with their curve-fits, since they may include a dark matter component which exists only to ‘fix’ their wrong solution.
- **Ignore the fact that a spiral galaxy is not an infinitesimally thin disk.** It has a thickness and a z-profile no matter how thin, and all measurements are actually an integral through the galaxy – not just ‘surface’ averages.
- **Analyze with the wrong equations.** If the measurements are integrated over z, then the equations predicting them must be as well.
- Correct your incorrect calculation by adding ‘**dark matter**’.

Basic Equations: Momentum and Gauss's Law are easily solved with many AI solvers. You really only need Gauss's equation for ϕ and $V_c(r)$, not its definition.

$$-\frac{V_c^2}{r} = -\frac{\partial \Phi}{\partial r}$$

Also it is common to define a '*circular* velocity', say V_c by using all the terms on the left-hand side except the time derivative; i.e.,

$$-V_c^2 = -V_\phi^2 + \left\{ \frac{\partial (r V_r^2)}{\partial r} + r V_r^2 \frac{\partial \ln \rho}{\partial r} + \frac{\partial (V_\phi V_r)}{\partial \phi} + [(V_\phi V_r)] \frac{\partial \ln \rho}{\partial \phi} + r V_r V_z \frac{\partial \ln \rho}{\partial z} - r \frac{\partial}{\partial r} \left(V_z^2\right) - r V_z^2 \frac{\partial \ln \rho}{\partial r} - r \frac{\partial V_z^2}{\partial r} \right\} \quad (20)$$

Except near the core region, the first term, V_ϕ^2 dominates.
Since the velocities are small compared to the speed of light, Gauss' gravitational law applies. In cylindrical polar coordinates it is given by:

$$\left[\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right] \Phi(r, \phi, z) = 4\pi G \rho(r, \phi, z), \quad (24)$$

where G is the universal gravitational constant.

Examples of how to create Dark Matter from nothing.

- Start with a specified density profile.
- Solve Gauss's equation for the gravitational potential.
- Solve the simplest momentum equation for the velocity squared.
- Solve for the profile as z=0. And also integrate it over z.
- All the data flattens out with increasing radius. But your solution evaluated at z= 0 drops as 1/r.

Example 1: Creating ‘Dark Matter’ in exponentially decaying density galaxies

$$\overline{\overline{\rho_{eg}}}(r,\phi,z) \;=\; \rho_o\,\left[exp(-z^2/2\sigma_z^2)\right]\; exp(-r/z_r)$$

$$\overline{\overline{V^2_{cge}}}(r,0) = G\,M_{ge}\,\frac{r^2}{2z_r^3}\left[I_0\left(\frac{r}{2z_r}\right)K_0\left(\frac{r}{2z_r}\right) - I_1\left(\frac{r}{2z_r}\right)K_1\left(\frac{r}{2z_r}\right)\right]. \quad (5$$

$$\begin{aligned}\langle V^2_{cge}(r)\rangle \;&=\; r\,\frac{\partial\langle\Phi_{ge}(r)\rangle}{\partial r} = r\int_{-\infty}^{\infty} dz\,\frac{\partial\overline{\overline{\Phi}}_{ge(r,z)}}{\partial r}\\ &=\; 2\,G\,M_{ge}\,[1-(1\,+\,(r/z_r))\exp(-r/z_r)] \qquad (53)\end{aligned}$$

$$\begin{aligned}4\,\pi\,G\,\langle\rho_{dm}(r)\rangle \;&=\; \frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{1}{z_m(r)}\langle\frac{r\,\partial\Phi_{ge}(r)}{\partial r}\rangle - \frac{r\,\partial\overline{\overline{\Phi}}_{ge}(r,0)}{\partial r}\right\}\\ &\rightarrow\; \left[\frac{2\,M_{ge}\,G}{z_r^3}\right]\left\{\left(\frac{z_r}{z_m}\right)\exp(-r/z_r) + \frac{1}{2\,(r/z_r)^3}\right\}\end{aligned}$$

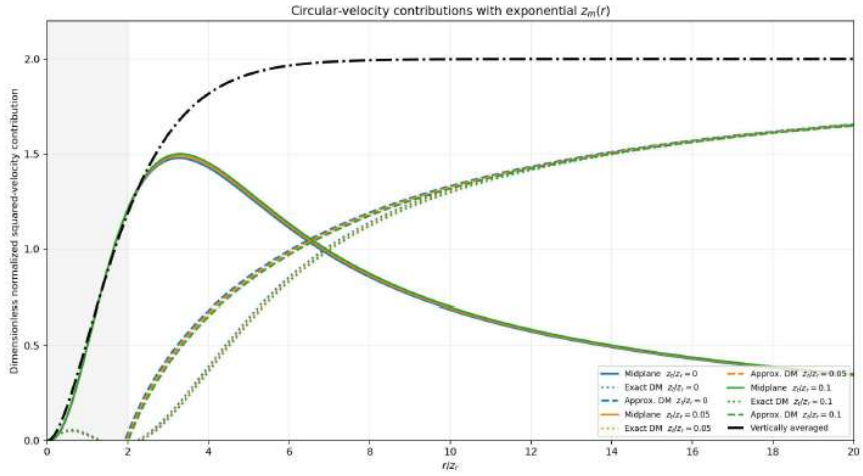


Figure 10: Generic plots of $\langle V^2_{cge}/GM_{ge}\rangle$ and (z_m/z_r) times $\overline{\overline{V^2_{ge}}}(r,0)/GM_{ge}$ and $\Delta V^2_{dmge}(r)/M_{ge}$for Gaussian-exponential density. This parallels the way astronomical data would be fitted to data. The values of z_t/z_r are $0, 0.05$ and 0.1. $z_m(r)/z_m(\infty) = 1 - exp(-Ar/z_r)$. $z_m(\infty)/z_r$ were calculated from equation 62. Also shown are the two ‘*dark matter*’ contributions (equations 91 and 95) which would have been inferred if the integration over z had not been included in the analysis.

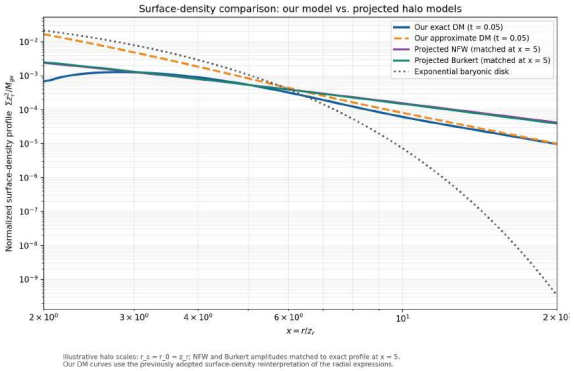


Figure 11: Plots showing the dark matter density profile, $\langle\rho_{dmge}(r)\rangle$ (equations 92 and 98) that would have been imagined to be true if analyzed by the centerline equations instead of the vertically integrated ones. Also shown are two commonly used empirical ‘dark matter halo’ profiles of Burkett [8] and NFW [58] which nearly parallel our approximate ‘*fake dark matter*’ profile.

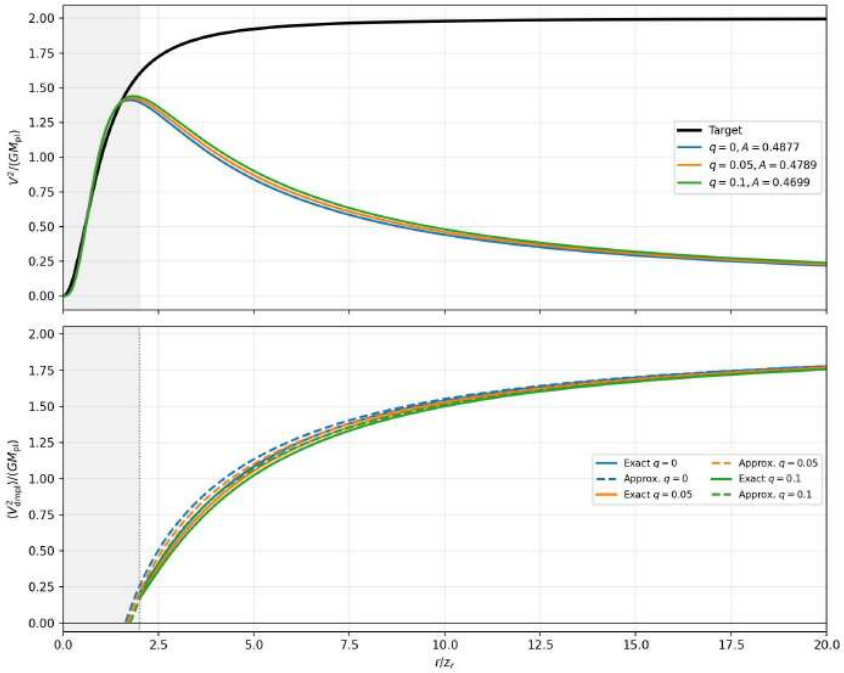
Example 2: Creating ‘Dark Matter’ in Power Law Density Galaxies

$$\overline{\overline{\rho}}_{pl}(r,z))\rangle \quad = \quad \rho_o \left\{1+(z/z_t)^2+(r/z_r)^2\right\}^{-5/2}$$

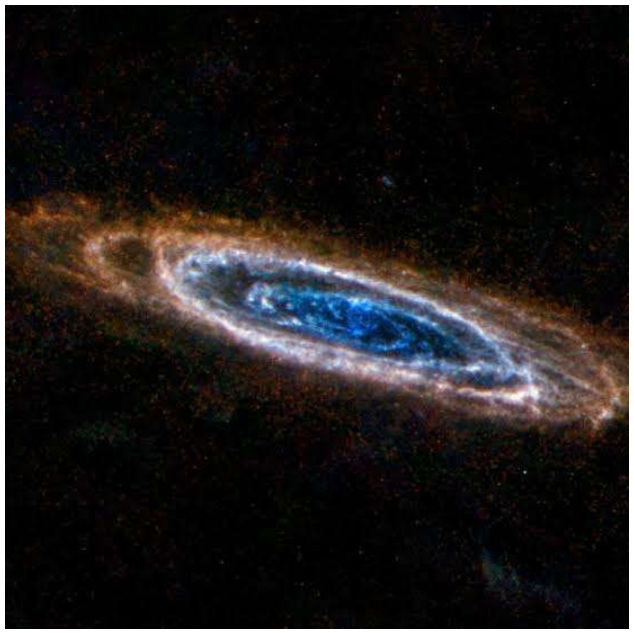
$$\overline{\overline{V^2_{pl}}}(r,0) \quad = \quad \frac{r\,\partial\overline{\overline{\Phi}}_{pl}(r,0)}{\partial r} = \frac{3\,G\,M_{pl}}{2}\,r^2\int_0^\infty du\frac{\left[1+r^2/(z_r^2+u)\right]^{-}}{(z_r^2+u)^2\sqrt{z_t^2+}}$$

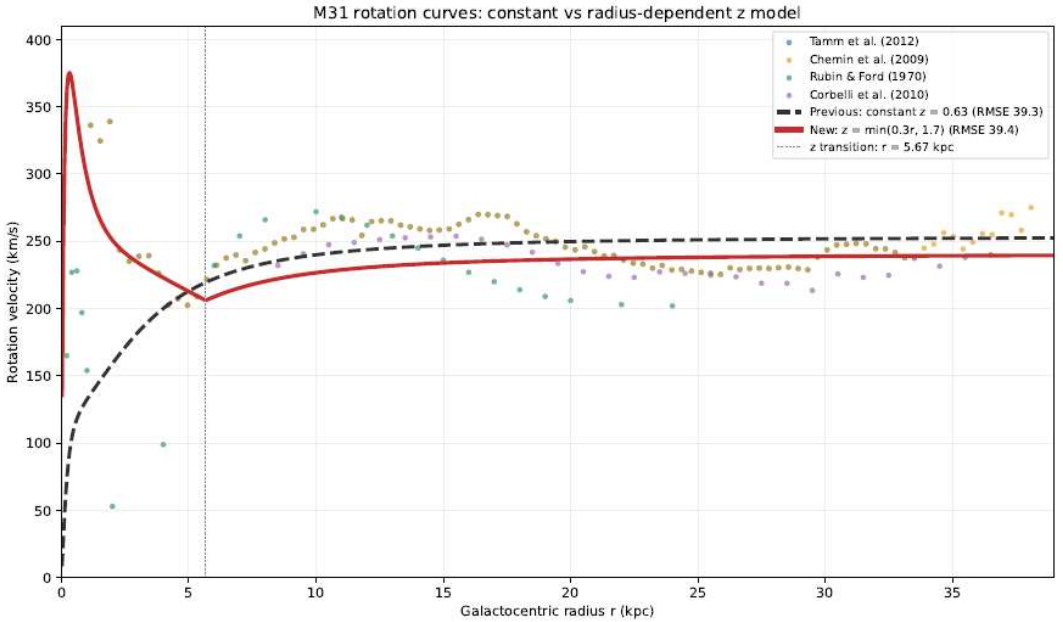
$$\langle V^2_{cpl}(r)\rangle = \langle\frac{r\,\partial\Phi_{pl}(r,z)}{\partial r}\rangle = \frac{8\pi G\,\rho_o\,z_r^2 z_t}{3}\frac{r^2/z_r^2}{[1+(r/z_r)^2]}$$

Figure 12: Upper: Generic plots of $\langle V^2_{cpl}/GM_{pl}\rangle$ and (z_m/z_r) times $\overline{\overline{V^2_{pl}}}(r,0)/GM_{pl}$ and $\Delta V^2_{dmpl}(r)/M_{pl}$for power law density. The values of z_t/z_r are $0, 0.05$ and 0.1. $z_m(r)/z_m(\infty) = 1 - exp(-Ar/z_r)$. $z_m(\infty)/z_r$ calculated from equation 79. This parallels the way data are usually treated. Lower: The ‘*dark matter*’ contributions from equations 97 and 99 that would have been inferred had the theory not been integrated over z.



Real data and vertically integrated solution for M31 (Andromeda). Note that when integrated over z it needs no dark matter





Real data and vertically integrated solution for M33 (Triangulum). When integrated over z it needs no dark matter.

Figure 16: Fit of velocity calculated from equation 135 to M33 velocity data of Corbelli and Salucci [18] using only the similarity parameters determined from the theory. z_m was modeled as linear from $0 < r < 1.5$ kpc and constant at $z_m = 0.59$ kpc for $r > 1.5$ kpc. z_r was chosen as 2. Contrast our result to Figure 1 which treated the same data as only azimuthally-averaged, not spatially-averaged. There is clearly no need for dark matter.

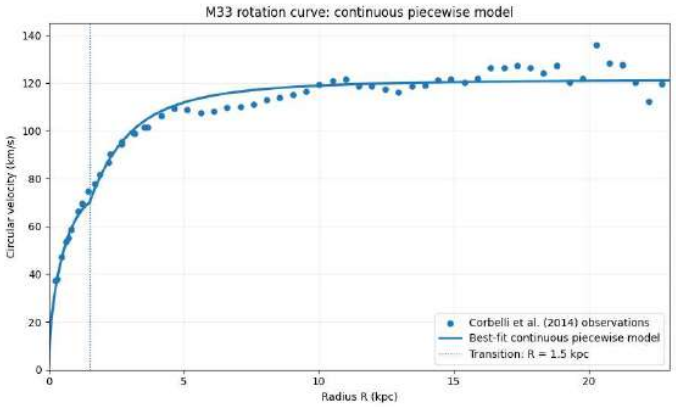
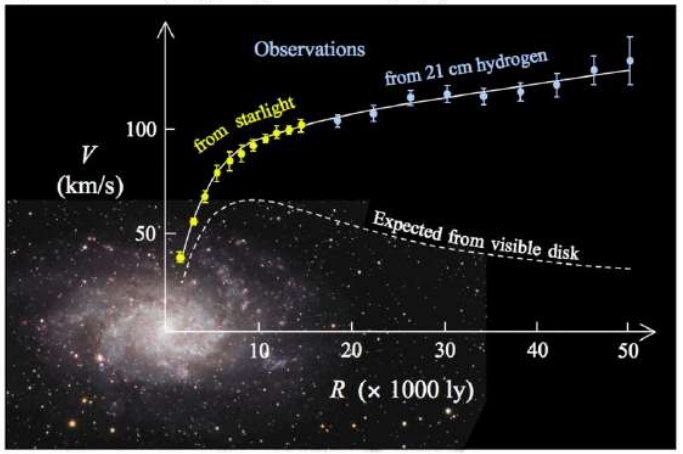
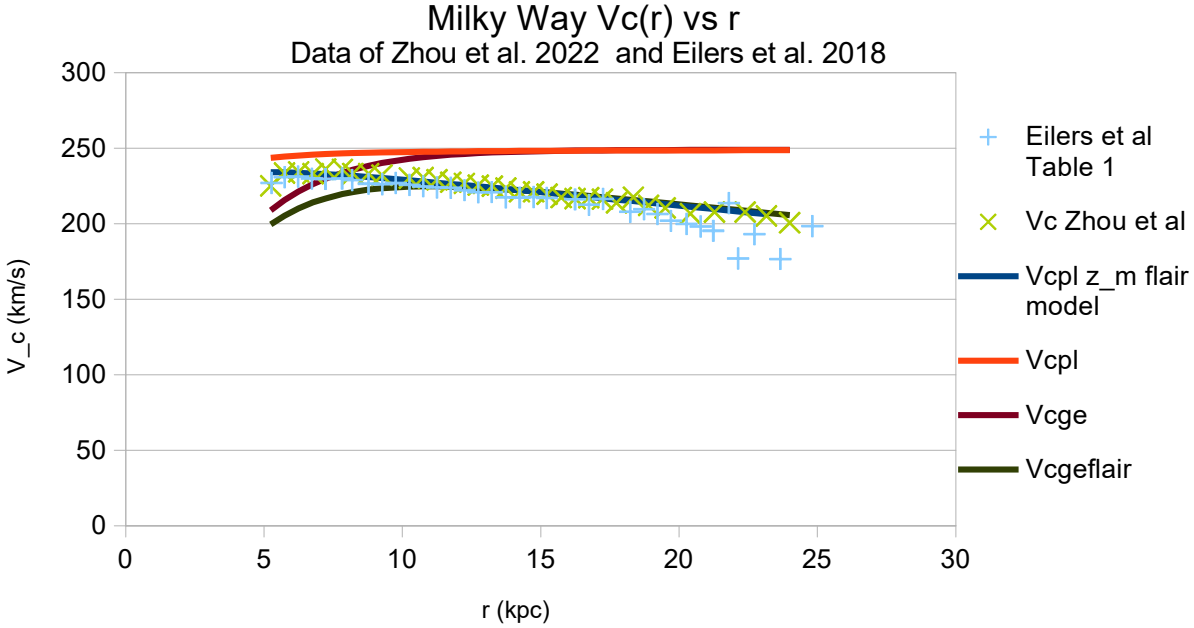


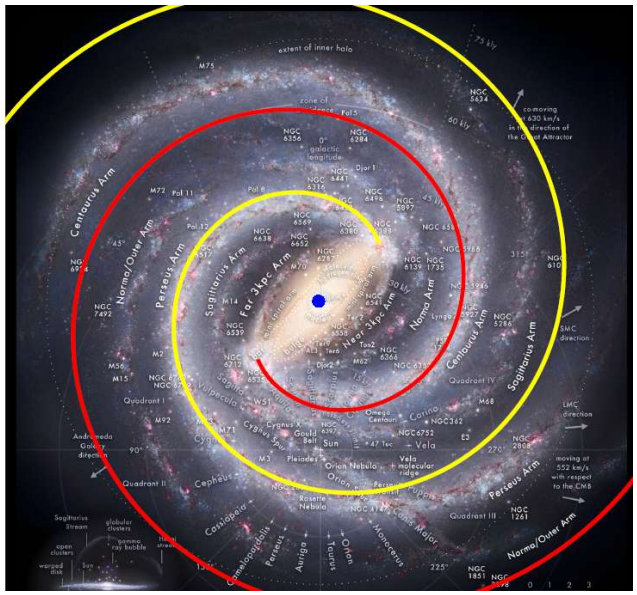
Figure 1: Galaxy M33 showing the visible matter, the measured azimuthal velocities and the theoretical calculation using the Gauss-Newton equations. The difference between '*expected*' and '*observations*' is usually attributed to dark matter' (Data from [18], Figure from [74].)



The Milky Way. We first presented this at the 2023 APS-DFD meeting in DC. It's taken us until now to figure out what everyone else was doing wrong.

Figure 5: Figure showing the streamlines computed from the velocities of Figure 1 of Eilers et al. [25] (reproduced as Figure 3 above) and super-imposed on the Milky Way image of [73]. Note that this figure was presented originally at the Gallery of Fluid Motion at the 2023 APS/DFD meeting in Washington DC.





Flared and unflared solutions. **There are no adjustable parameters in this plot**, **nor is there any need for dark matter or dark energy.** The flare is what was measured and result of being in the galaxy and looking outwards with a cone of acceptance.

What did they really measure?

$$\overline{\overline{\overline{V^2_{\alpha\,meas}}}}(r;\Delta z(r)) \;=\; \overline{\left[\frac{1}{\overline{\overline{N_{col}}}(r)}\right]}\;\overline{\left[\overline{N_{col}}(r)\right]}\left\{\frac{1}{\Delta z}\int_{-\Delta(r)/2}^{\Delta z(r)/2} dz'\;\overline{\overline{\overline{V^2_{\alpha *}}}}(r,z')\right\} \tag{128}$$

Clearly the term in curly brackets, $\overline{\overline{\overline{V^2_\alpha}}}(r,z)$, is just the star velocities averaged azimuthally and integrated over the interval from $z = -\Delta z/2$ to $z = +\Delta z/2$. If $\Delta z > z_t$ by a lot, the integral is just the definition of the spatially averaged velocity, $\langle V^2_\alpha(r)\rangle$. This is our model. On the other hand, if $\Delta z << z_t$, it can approximated as Δz times its value at the centerline ($z = 0$); i.e., $\overline{\overline{\overline{V^2_{\alpha *}}}}(r,0)$. This has been the basis of all previous analyses.

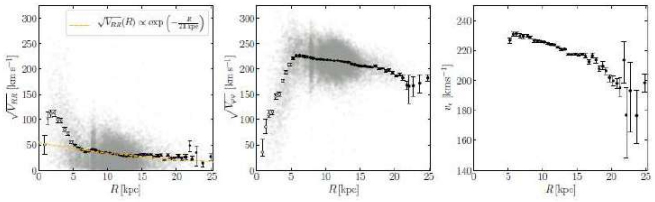


Figure 1. Radial profiles of the components of the velocity tensor $\sqrt{V_{RR}}$ (left panel), $\sqrt{V_{\varphi\varphi}}$ (middle panel), and the circular velocity v_c (right panel). The small grey dots show individual stars, whereas the black data points show the ensemble averaged values with uncertainties determined via bootstrapping with 100 samples. We exclude all stars within a distance of $R < 5$ kpc from the Galactic center (open data points) from our analysis due to the presence of the Galactic bar.

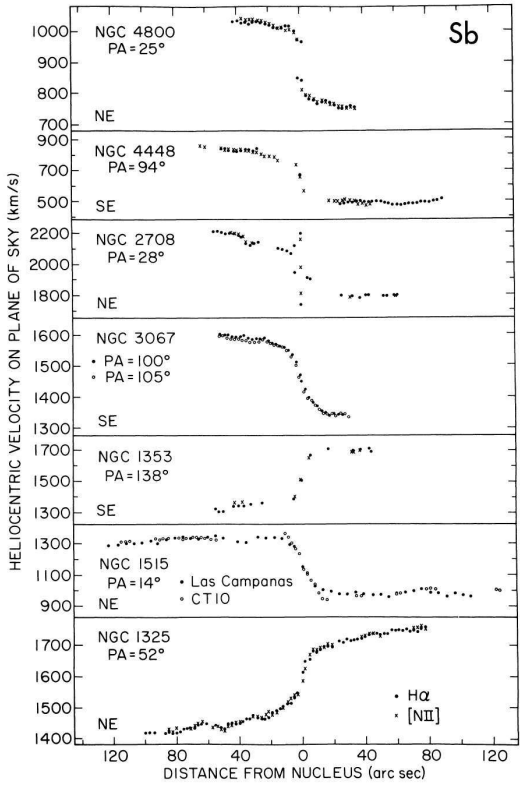


FIG. 2a

FIG. 2.—Heliocentric line-of-sight velocities on the plane of the sky as a function of angular distance from the nucleus for 23 Sb galaxies

So which is right? Approximating $V_c(r,z)$ using the centerline (z = 0), or treating it as a spatially averaged quantity integrated over both z and ϕ?

- Actually both are right – depending on how thick the galaxy is relative to the resolution in z.

Since we assume $\Delta z << z_t$ we can approximate the integral by the first two terms in its Taylor expansion in z; i.e.,

$$\int_{-\Delta(r)/2}^{\Delta z(r)/2} dz' \, \overline{\overline{V_\alpha^2}(r,z')} \approx \Delta z \, \overline{\overline{V_\alpha^2}(r,0)} + \frac{(\Delta z)^3}{12} \frac{\partial^2 \overline{\overline{V_\alpha^2}(r,0)}}{\partial z^2} + \cdots \qquad (137)$$

where we have assumed the velocity profile to be symmetric in z and neglected the first derivative term.

So in this limit the measured velocity is exactly what seems to have been assumed by the astronomical community for years; i.e.,

$$\overline{V_{\alpha\,meas}^2(r;\Delta z(r))} = \overline{\left[\frac{1}{\overline{\overline{N_{col}}(r)}}\right]} \, \overline{\left[\overline{N_{col}}(r)\right]} \left\{ \overline{\overline{V_\alpha^2}(r,0)} + \frac{(\Delta z)^2}{12} \frac{\partial^2 \overline{\overline{V_\alpha^2}(r,0)}}{\partial z^2} \right\} \qquad (138)$$

For the $\Delta z >> z_t$, the integral over z is effectively infinite. So the measured velocity is:

$$\overline{V_{\alpha\,meas}^2(r;\Delta z(r))} \approx \overline{\left[\frac{1}{\overline{\overline{N_{col}}(r)}}\right]} \, \overline{\left[\overline{N_{col}}(r)\right]} \left\{ \frac{1}{\Delta z} \int_{-\infty}^{\infty} dz' \, \overline{\overline{V_\alpha^2}(r,z')} \right\} \qquad (129)$$

$$\approx \overline{\left[\frac{1}{\overline{\overline{N_{col}}(r)}}\right]} \, \overline{\left[\overline{N_{col}}(r)\right]} \frac{\langle V_\alpha^2(r)\rangle}{\Delta z}$$

Clearly in this limit, the measured velocities are exactly equal to the spatially-averaged velocity, $\langle V_\alpha^2(r)\rangle$.

The approximation on the left is justified by arguing that the galaxy profile is very thin. But that justifies the assumption on the right, since infinitely thin would require infinitesimal resolution to resolve the peak. Only the LEFT assumption requires dark matter to fill the measured vs theory velocity gap. The RIGHT assumption needs none!

Take the dark matter challenge!

- use any AI version, but don't believe anything else it tells you :-)

Signed copy of our paper if you get it right!
And lots of fun with your astronomer friends. :-)

Assume-dimensional density is given by the familiar choice:

$$\rho_{ee}(r,\phi,z) = \rho_{oee}\,(1+cos(m\phi))\,[exp(-|z|/z_t]\;exp(-r/z_r)$$

where z_z and z_r are usually assumed to be constants and m is an integer.

Compute 'dark matter' density and correction to the velocity-squared.
Extra credit if you find the dependence on m.

Summary

- All the evidence (both measurements and theory) suggests that the velocity measurements in spiral galaxies are averaged over the galaxy thickness.
- And when analyzed by the correct **z-integrated** equations, there is no need for either dark matter nor dark energy to describe them.
- The work described herein is in the latest version of a paper under review by Physical Review F submitted in Dec 2004. Although provisionally accepted in May 2005, **we** have delayed publication this past 18 months in order to derive and include these results. They were non-trivial to derive by hand. And given their propensity for errors, even a challenge for Claude and Chat-GPT, thus necessitating both methods of solution.
- The latest draft should be available on our website (www.turbulence-online.com) by the end of December (2026).