Could time be logarithmic?

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November 26, 2016

Abstract

This paper explores the consequences of the hypothesis that time might be logarithmic. Two versions of logarithmic time are explored: the first where the physical laws are written in fixed space, and second where space itself is expanding.

1 Introduction

The description of a universe evolving in '*epochs*' of logarithmic time is familiar to every cosmologist (see Table 1 taken from Wikipedia [1]). There appears to be no evidence that it has ever been previously considered that logarithmic time should be applied to physical laws. This paper simply explores the implications and consequences if our physical laws should have been written using log-time as well.

The idea of time varying logarithmically has been present in turbulence theory for some time, but only recently recognized [2]. There it is the growth of length scales with time in the decaying homogeneous turbulence which dictates that turbulence evolves in logarithmic time increments. In effect 'time' slows down as the turbulence decays. Originally it was thought that perhaps such 'solutions' applied to the universe as well, and indeed they might. If so, what we think we perceive as the universe expanding might simply be the scales growing – as in the turbulence solutions. But this paper explores an entirely different idea – that time itself might be logarithmic. And that perhaps the laws of physics should have been written using logarithmic time. Note that we are not talking about simply a coordinate transformation of existing equations and laws here, but instead completely new equations and laws. But equations and laws close enough to those we have believed for over a century now that we were fooled into thinking they were an accurate descriptions of nature.

The most important difference in a log-time universe from a linear time one would be that we must modify our definition of mass. And of course velocity

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Log-time	Seconds after the Big Bang	Period
-45 to -40	10^{-45} to 10^{-40}	Plank Epoch
-40 to -35	10^{-40} to 10^{-35}	
-35 to -30	10^{-35} to 10^{-30}	Epoch of the Grand
-30 to -25	10^{-30} to 10^{-25}	Unification
-25 to -20	10^{-25} 10 10^{-20}	
-20 to -15	10^{-20} to 10^{-15}	
-15 to -10	10^{-15} to 10^{-10}	Electroweak Epoch
-10 to -5	10^{-10} 10 10^{-5}	
-5 to 0	10^{-5} to 10^{-0}	Hadron Epoch
0 to +5	10^{0} to 10^{5}	Lepton Epoch
+5 to + 10	10^{5} to 10^{10}	Epoch of Nucleosynthesis
+10 to +15	10^{10} to 10^{15}	Epoch of Galaxies
+15 to +20	10^{15} to 10^{20}	

Table 1: Each row is defined in seconds after the Big Bang epochs of logarithmic time in cosmology with earliest at the top. The present time is approximately 4.3×10^{17} seconds after the Big Bang.

and acceleration as well. This will be seen to have no measurable consequences over the span of our human existence. But it has great consequences for how we interpret the results of applying our physical laws to astronomical observations of events that happened long ago. In particular, mass will appear to us to be missing, even when it is not. Also the universe will appear to be expanding, even if it is not.

This paper is organized in three parts. Part I defines logarithmic time and its relation to absolute time (as measured from a beginning). Part II looks at the consequences for our physical laws if they should have been written as log-time but in fixed space variables. And Part III considers how things might be different if we had to write our laws in both log-time and a spatially changing coordinates. An attempt has been made to make this as understandable as possible to those with even a high school and early college level physics background.

Part I: What is logarithmic time?

2 The hypothesis of logarithmic time

It is hypothesized that a *cosmos time*, say τ, might describe the cosmos and all physical laws governing it. It is further postulated that this *cosmic time* varies as the logarithm of t/t_o; i.e.,

$$\tau = \ln t/t_o \tag{1}$$

where t is the time (or *absolute time*) measured in linear increments since the beginning at $t = 0$. t_o is any convenient time scale and sets the units of t (e.g., seconds or years). The exact beginning chosen is arbitrary, but for our purposes the Big Bang is probably acceptable, although no quantum mechanical effects will be considered (although they could be).

3 Would we have noticed the difference?

Any cosmic (or logarithmic) time difference between two cosmic times, say τ_p and $\tau_p + \delta\tau$, can expressed in linear time increments by the difference of their Taylor series expansions; i.e.,

$$\delta\tau = \ln[(t_p + \delta t)/t_o] - \ln[t_p/t_o] \quad (2)$$
$$= \ln[(t_p/t_o)(1 + \delta t/t_p)] - \ln(t_p/t_o) \quad (3)$$
$$= \ln(t_p/t_o) + \ln(1 + \delta t/t_p) - \ln(t_p/t_o) \quad (4)$$
$$\approx (\delta t/t_p) + (\delta t/t_p)^2 + \cdots \quad (5)$$

where t_p and $t_p + \delta t_p$ are the corresponding absolute times.

It is estimated that there have been approximately 13.7 billion years ($t_p \approx 4.3 \times 10^{17}$ s) since the Big Bang. Mankind has only been on the earth for approximatly 250,000 years. So even if we had been keeping careful track since then the differences we would have noticed between the hypothesized cosmic time and linear time would have been $\delta t/t_p \approx 2.5 \times 10^5/13.7 \times 10^{12} \approx 3.4 \times 10^{-8}$. And the differences we would have needed to observe to discover a discrepancy are the square of this, or of order 10^{-15}. But we have been doing mechanics for only the past 500 years, so even had we started measuring carefully at Galileo, $\delta t/t_p \approx 500/13.7 \times 10^{12} = 3.6 \times 10^{-11}$. So the leading error term would have been of order 10^{-21}, and clearly beyond our ability to distinguish from experimental data alone.

So the point is that time could have been logarithmic all along, but we might have never suspected nor noticed.

4 The natural laws of physics

The whole idea of a **natural law** is that it expresses by equations experimental observations which cannot be derived on more fundamental grounds. Examples for us are Newton's Law (as modified by Einstein), and Maxwell's equations, and Einstein's Law that the speed of light is constant.

So the question for us is this: *If all of the natural laws of physics should have been written in the hypothesized logarithmic time (or* **cosmic time***), would we have ever been able to tell the difference?*

Clearly only by looking far far back in time (on cosmic scales) might we have seen any measurable differences. Our laws might have appeared not to work –

something would have been amiss. The equations simply might not have added up when applied to the data.

In the entire history of physics we have generally been slow to recognize when there is a problem with our fundamental equations.[3] We have been more likely to invent 'fixes' to make them work – ether, phlogiston fictitious energy, fictitious mass, fictitious extra terms in equations which have no physical justification.

Only when the number of 'fixes' needed proliferated and the incongruities became intolerable did the paradigm shift follow.[3]. Ultimately it is the changing of the equations themselves which triumphs. It is suggested that this might be the case here. The following sections explore the consequences the laws of mechanics, gravity, and radiation.

Part II: Velocity, acceleration, momentum and energy in a gravitational field

5 Newton's 'law' rewritten for logarithmic time

Fundamentally Newton's law is a definition of mass, say m. And mass is measured by its inertia, force divided by acceleration, where traditionally accelerations are measured in linear time increments. Both force and acceleration are measurable. In the absence of either, mass is not.

If he had believed time to be logarithmic, Newton could have equally written his law as:

$$\vec{f} = m^* \frac{d\vec{v}}{d\tau} = m^* \frac{d\vec{v}}{d\ln(t/t_o)} \tag{6}$$

where m^* is our new definition of mass (say **cosmic mass**) and $\vec{v}$ is the velocity, the precise definition of which we reconsider later.

But it follows immediately that:

$$\vec{f} = m^* \frac{d\vec{v}}{d\ln t/t_o} = [m^* t] \frac{d\vec{v}}{dt} \tag{7}$$

Recall that t is the absolute time measured linearly since the beginning of time, and m^* is presumed time-independent. So even if our definition of velocity does not need to change, *the mass m, we would have inferred by applying a force and measuring the acceleration in linear increments is in reality* **time-dependent**. And the farther we look back in time, the smaller our mass, m, would appear to be. Any attempt by us to describe its dynamics with the usual Newton equations would find them unbalanced with too little mass.

6 What about velocity?

Velocity itself is problematic, since it too depends on time derivatives – linear ones according our history. But suppose it too should be computed in logarithmic time increments. Tben we should be taking logarithmic time derivatives of spatial position change.

For example if a body is moving though space and its coordinates can be given by $\vec{x} = \vec{x}_p(t) = \vec{X}_p(\tau)$, then the proper **cosmic velocity**, say $\vec{V}$, would be given by:

$$\vec{V} = \frac{d\vec{X}_p}{d\tau} = t\frac{d\vec{x}_p}{dt} = t\ \vec{v} \tag{8}$$

where again t is absolute time and $\vec{v}$ is the velocity we would have computed with normal linear time increments. But in our post-Newton era we would have never known the difference, even if it mattered for physics, because t is effectively constant (to one part in 10^{11} or less) during the span of our human existence!

An interesting consequence of our definitions of cosmic mass and velocity is that kinetic energy remains the same; i.e, $m|\vec{v}|^2/2 = m^*|\vec{V}|^2/2$. And since the speed of light will need to be measured in cosmic time increments as well; $mc^2 = m^*c_*^2$ as well, where $c_* = t\ c$. As noted above, distinguishing whether either c and c^* is constant is well beyond our ability to determine since the variation of t itself is so small. If one appears to be constant (as we have believed c is), then both will appear to be. And vice versa.

7 Accelerations in logarithmic time

It follows that the logarithmic acceleration would be given by:

$$\vec{A} = \frac{d\vec{V}}{d\tau} = \frac{d^2\vec{X}_p}{d\ln(t/t_o)^2} \tag{9}$$

$$= t^2\frac{d^2\vec{x}_p}{dt^2} + t\frac{dx_p}{dt} \tag{10}$$

$$= t^2\left\{\vec{a}_p + \frac{1}{t}\vec{v}_p\right\} \tag{11}$$

If we assume that $|\vec{v}_p|$ is bounded by the speed of light, say $c = 3 \times 10^8$ m/s, and in our human era of physics $t > 10^{17}$s, then contribution of the last term is of order $10^{-9}\ m/s^2$ or less. The last term would have most probably have not been noticed, even if it were measurable.

So the conclusion is that if time were indeed logarithmic, then our replacement for Newton's Law probably should be:

$$\vec{f} = m^*\frac{dV_p}{d\tau} = m^*\frac{d_p^X}{d\tau^2} \tag{12}$$

$$= \quad m^* t^2 \left\{ \frac{d^2 \vec{x}_p}{dt^2} + \frac{1}{t} \frac{dx_p}{dt} \right\} \tag{13}$$

$$\approx \quad [m^* t^2] \; \vec{a}_p \tag{14}$$

Thus for all times not close to the very beginning of time, the mass, m, obtained on earth is approximately the true cosmic mass times the age of the universe squared; i.e.,

$$m \approx m^* t^2 \tag{15}$$

Clearly the farther back we travel in time for our observations, the more important the departures from the classical Newton's law become. Obviously any attempt to balance the equations using observations of events a long time ago would have appeared to have mass missing – at least if we were inferring mass, m, by applying Newton's law. And the farther back in time we look, the worse it would seem to be.

8 Gravity and mass

There is nothing fundamental about gravity which changes if we change our equations to reflect a dependence on logarithmic time. We would need, however, to change the definition of mass and the gravitational constant to reflect the differences. It is not gravity which is changing, but the masses we associate with it; e.g., m^* instead of m. Also there is no reason to assume our previous experiments to determine G are incorrect, only that the definitions of accelerated mass we have associated with a given gravitational attraction need to change.

Newton's gravitational law is commonly written as:

$$F = G \frac{m_1 m_2}{r^2} \tag{16}$$

where $G = 6.67408 \times 10^{-11} \; m^3/kg \; s$ to one part in 4.7×10^{-5}.

If time were logarithmic, then $m = m^* t^2$. And we would rewrite the gravitational law as:

$$F = G^* \frac{m_1^* m_2^*}{r^2} \tag{17}$$

So we need to include only the factor of t_p^4 into our definition of G^* where t_p is the absolute time the measurement of G was made.

Thus our best estimate for G^* is

$$G^* = G \; t_p^4 \tag{18}$$

The estimated age of the universe in linear seconds is approximately $t_p \approx 4.320432 \times 10^{17}$ seconds, so:

$$G^* = G t_p^4 \approx 3.48425^{70} \times 6.6705 \times 10^{-11} = 2.325 \times 10^{62} \; m^3 s^3/kg \tag{19}$$

Note that the units of cosmic mass, m^*, are kg/s^2. Clearly given the size of the numbers, there must be a better choice of units. Note also that since G enters the Planck scale definitions, introduction of G^* will change some of these as well.

Note also, as we shall see in Part III that in a expanding universe it might be more logical to write the inverse square law in scaled coordinates. This would further change the definitions as follows:

$$F = G^{**}\frac{m_1^* m_2^*}{\eta^2} \tag{20}$$

where G^{**} incorporates both the factor of t_p^4 and δ^{-2}; i.e.,

$$G^* = G\ \frac{t_p^4}{\delta^2}. \tag{21}$$

So in summary, there appears to be nothing fundamentally different about gravity. BUT, as we shall see below the consequences of the different definitions of mass, m^*, and the Gravitational constant, G^*, have enormous consequences for the application of the gravitational laws at different cosmic times.

9 Energy and the virial theorem

It should be clear from the above that in a system where time is logarithmic, we must be willing to reconsider the basic ideas of what is invariant and what is not. Of particular importance for cosmology is the Hamiltonian defined as $H = \Sigma p_i r_i$ where p_i is the momentum of the i-th member of the collection of masses, r_i is its position, and the summation is over all i.?

Following the proof outline of Thayer Watkins in notes at San Jose University [4], we define

$$H^* = \Sigma P_i^* r_i \tag{22}$$

where as before the subscript i denotes the i-th member of the system, m_i^* is its cosmic mass, r_i is its position relative to it's center of mass, $P_i^* = m_i^* V_i$ is the newly defined momentum and $V_i = dr_i/d\tau$. For now we only consider the forms when the coordinate system is *not* expanding.

First take the logarithmic derivative to obtain:

$$\frac{dH^*}{d\tau} = \Sigma r_i \frac{dP_i^*}{d\tau} + \Sigma P_i^* \frac{dr_i}{d\tau} \tag{23}$$

But $F_i = dP_i^*/d\tau$. So

$$\frac{dH^*}{d\tau} = \Sigma r_i F_i + \Sigma P_i^* \frac{dr_i}{d\tau} \tag{24}$$

And since $P_i^* = m_i^*(dr_i/d\tau)$ the last term reduces to

$$\Sigma P_i^*(dr_i/d\tau) = \Sigma m_i^*(dr_i/d\tau)^2 = \Sigma m_i^* V_i^2 \tag{25}$$

This is just twice the kinetic energy of the system , say $2K$; so,

$$\frac{dH^*}{d\tau} = \Sigma F_i r_i + 2K \tag{26}$$

(Interestingly, as noted above, since $m \approx m^* t^2$, K is the kinetic energy in both logarithmic and linear time variables).

Second we assume the forces acting to be conservative so $F_i = -d\Phi/dr_i$ where Φ is the gravitational potential field. Substituting yields:

$$\frac{dH^*}{d\tau} = -\Sigma r_i \frac{d\Phi}{dr_i} + 2K \tag{27}$$

If the system is cyclical, like two galaxies in rotation, then averaging over a period (in τ) yields $\langle dH/d\tau \rangle = 0$. So the average kinetic energy is just half the potential energy; i.e.,

$$\langle K \rangle = \frac{1}{2} \langle \Sigma r_i \frac{d\Phi}{dr_i} \rangle \tag{28}$$

This looks exactly like the result for linear time. Only the gravitational potential is different – both because of the definition of mass and G^*.

Third we need to compute the average potential energy of n galaxies. The average gravitational potential energy for two identical galaxies separated by distance R is $G^* m^* 2/R$. If there are n galaxies, there are $n(n-1)/2$ pairs of galaxies. So the average kinetic energy of n identical galaxies is $nm^* \langle V^2 \rangle /2$. And this must be equal to half the average potential energy, so:

$$n \frac{1}{2} m^* \langle V^2 \rangle = \frac{1}{2} (n)(n-1) \frac{1}{2} \left[G^* \frac{m^{*2}}{R} \right] \tag{29}$$

Finally,solving for m^* yields:

$$m^* = \frac{2 \langle V^2 \rangle R}{G^* (n-1)} \tag{30}$$

Thus the total mass of the n-galaxies in the cluster is:

$$nm^* = \left[\frac{2n}{n-1} \right] \frac{\langle V^2 \rangle R}{G^*} \tag{31}$$

At first glance this appears to be the previous result for linear time. But it is not, since the definition of mass, m^*, velocity, V, and gravitational constant, G^*, are different.

We can put this result in terms of our previous definitions using $m^* = mt^{-2}$, $V = tv$ and $G^* = Gt_p^4$. The result is:

$$nmt^{-2} = \left[\frac{2n}{n-1} \right] \frac{t^2 v^2 R}{G t_p^4} \tag{32}$$

or

$$m = \left[\frac{2}{n-1}\right]\left[\frac{v^2 R}{G}\right]\left[\frac{t}{t_p}\right]^4 \tag{33}$$

So if time is logarithmic, then the mass at time t will be over-estimated by a factor of $(t_p/t)^4$ using the linear time analysis. This seems to correlate very well with observations in rotating galaxies where dark matter is needed to account for the 'missing mass' from astronomical observations.

10 So how could the astronomers have gotten the correct answer?

Clearly astronomers have come up with much smaller estimates of the mass of galaxies than that estimated from gravitational considerations. Why? And how do we know that their methods will come up with the same answer if time is logarithmic?

When astronomers estimate the mass of galaxies (or collections of them), they do so by counting up the stars, accounting for their luminosity, and comparing them to them to our sun.

Question: But how do they determine the mass of the sun? Answer: they apply the very laws we are questioning to calculate it from the mass of the earth.

Question: But how do they know the mass of the earth? Answer: they apply the classical *universal gravitation law* and calculate from the measured gravitational acceleration; i.e.,

$$M_e = \frac{R^2\ g}{G} \tag{34}$$

where R is radius of the earth and g is the gravitational acceleration.

The big difference is that the time used to convert mass M_e to cosmic mass M_e^* is the present time, t_p. And since all the inferences about the actual mass of stars is assume to be proportional to the sun, it is also t_p which should be used to convert those masses, no matter how far back in time the observations were made. This is very different from the gravitational considerations above where the absolute time of the event had to be used since the calculations were dynamic.

As a final note, if the problem were with the equations, especially as applied to spinning galaxies, then that is where the missing matter (or *dark matter*) would be needed – not uniformly spread throughout the universe. That seems to be consistent with their observations.

11 Propagation of radiation at constant phase

Let's consider what happens if an oscillator radiates into space with logarithmic time, $\tau = \ln(t - t_{osc})/t_o$ instead of the usual linear time. Its location is at fixed $\vec{x}_{osc}$ and it begins at time t_{osc}. The phase at any time and location would be given by:

$$\phi = \vec{\kappa} \cdot (\vec{x} - \vec{x}_{osc}) - \omega_*(\tau - \tau_{osc}). \tag{35}$$

$\vec{k} = \nabla_{\vec{x}}\phi$ is the wavenumber and $\omega^* = -d\phi/d\tau$ are the wavenumber vector and frequency in our logarithmic time space.

The important question for us on earth is: what do we see in our linear time, linear space coordinates? The wavenumber vector is of course the same. But the 'frequency' and 'phase velocity' are given respectively as $\omega = -d\phi/dt$, and $c = \omega/|\vec{k}|$.

It follows immediately that the frequency we see is given by:

$$\omega = \frac{d\phi}{dt} = \frac{\omega^*}{t} \tag{36}$$

Equation 36 clearly implies a red-shift. And the farther away and earlier it began, the greater the shift. **In fact most interestingly, it implies a red-shift even if that is no expansion at all!** So a red-shift alone does not imply expansion, at least if time were logarithmic.

The phase velocity seen in our earthly linear-time system would be:

$$c = \frac{\omega^*}{k^*}\left[\frac{1}{t}\right] = c^*/t \tag{37}$$

Or $c^* = c\ t$. If $\omega^*/k^* = c^* = \mathit{constant}$ then c measured by us would be time dependent. But expanding $1/t$ about any time interval in today's epoch means any discrepancies noted would be of order $\delta t/t_p$, and as noted above well below our ability to measure it. It might be noted that since we now define the standard meter to be exactly the distance traveled by the speed of light in a given time, then that definition makes it impossible to see any effect at all.

12 Maxwell's equations

Maxwell's equations (in Gaussian units) are given by [5]:

$$\nabla \cdot \vec{E} = 0 \tag{38}$$

$$\nabla \cdot \vec{B} = 0 \tag{39}$$

$$\nabla \times \vec{B} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{40}$$

$$\nabla \times \vec{E} = \frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{41}$$

But $(1/c)\partial/\partial t$ is exactly $(1/c_*)\partial/\partial\tau = (1/c^*)\partial/\partial \ln t/t_o$ since $c^* = t\ c$. So Maxwell's equations in log-time variables are exactly the same in either set of time variables; i.e.,

$$\nabla \cdot \vec{E} = 0 \tag{42}$$

$$\nabla \cdot \vec{B} = 0 \tag{43}$$

$$\nabla \times \vec{B} = -\frac{1}{c^*}\frac{\partial \vec{E}}{\partial \tau} \left(= -\frac{1}{c^*}\frac{\partial \vec{E}}{\partial \ln t/t_o} \right) \tag{44}$$

$$\nabla \times \vec{E} = \frac{1}{c^*}\frac{\partial \vec{B}}{\partial \tau} \left(= \frac{1}{c^*}\frac{\partial \vec{B}}{\partial \ln t/t_o} \right) \tag{45}$$

This is quite remarkable really!

13 Special relativity

We have seen in the preceding section that Maxwell's equations have an identical form in both linear time and log time variables. So aside from the fact that most of the relativistic equations of physics need to be rewritten in terms of cosmic time derivatives, nothing appears to change very much. But we have to be careful to make sure we distinguish between absolute time and time differences.

One reason for this is that the Minkowski metric is unchanged by the transformation to log-time. This is easy to see from the fact that $c^* = t\ c$ and $d\tau = d\ln t/t_o = dt/t$. So

$$dx^2 + dy^2 + dz^2 - c^2dt^2 = dx^2 + dy^2 + dz^2 - {c^*}^2 d\tau^2 \tag{46}$$

The same is true for the Lorentz transformation since $v^2/c^2 = V^2/{c^*}^2$. The only difference is that c^* is presumed constant instead of c. And Einstein's law of physics that the speed of light is constant is similarly modified so that it is c^* that is constant.

Finally we note that the important relativistic relationships (see Feynman vol I, Chap 16[6]),

$$E^2 - P^2c^2 = m_oc^2 \tag{47}$$

$$Pc = Ev/c \tag{48}$$

are also preserved in log-time coordinates. I.e.

$${E^*}^2 - {P^*}^2{c^*}^2 = {m^*}_o{c^*}^2 \tag{49}$$

$$P^*c^* = E^*v^*/c^* \tag{50}$$

Note that there may still be some question about how these apply to the stretched coordinates considered below. In particular, is it reasonable to apply to this very special coordinate system the hypothesis that laws of nature should be independent of coordinate system?

Part III: An expanding universe?

14 Accounting for an expanding universe

The computation of velocities and accelerations above do not account for any effects that might arise if the coordinate system we need to describe space might be expanding. Or that the universe itself might be expanding. Either way, we can examine this possibility by defining a scale length, say $\delta(t)$, that is a time-dependent. Note that we have used t instead of τ, but either is acceptable.

Now it makes sense to define our physical laws using both logarithmic time and our expanding coordinate system. So we define a position within it to be given by $\vec{\eta} = \vec{x}/\delta(t)$ and a displacement field to be defined by $\vec{\eta}_p = \vec{x}_p(\vec{\eta}, \tau)/\delta(t)$.

Now the logarithmic velocity in this field would be given by:

$$\vec{V}_p = \frac{d\vec{\eta}_e}{d\tau} \tag{51}$$

$$= \left[\frac{t}{\delta}\right] \left.\frac{\partial \vec{x}_p}{\partial t}\right|_t - \vec{\eta}_p \left[\frac{t}{\delta}\frac{d\delta}{dt}\right] \tag{52}$$

Or putting it in linear- time, linear-space variables, the velocity $\vec{v}_p = \partial \vec{x}_p/\partial t|_t$ would be given by:

$$\vec{v}_p = \left[\frac{\delta}{t}\right] \vec{V}_p + \left[\frac{d\delta}{dt}\right] \vec{\eta} \tag{53}$$

The farther away in η-space we view things, the faster they will appear to be moving, even if there is no relative velocity at the same place (i.e., $\vec{V}_p = 0$). In fact, it does not matter where we put the origin for η, the result will be the same everywhere. Locally (i.e., near $\eta = 0$), the two velocities, $\vec{v}_p$ and $\vec{V}_p$ are simply proportional to each other by the factor δ/t. Obviously the special case for $\delta \propto t$ is of great interest.

Now the acceleration in our four-dimensional expanding system can be similarly computed by defining it to be:

$$\vec{A}_p = \frac{d\vec{V}_p}{d\tau} \tag{54}$$

$$= \left[\frac{t^2}{\delta}\right] \frac{d^2\vec{x}_p}{dt^2} + \left[\frac{t}{\delta}\right] \frac{d\vec{x_p}}{dt} - \left[\frac{t^2}{\delta^2}\frac{d^2\delta}{dt^2}\right] \frac{d\vec{x_p}}{dt} - \eta_p \left[\frac{d^2 \ln\delta}{d(\ln t/to)^2}\right] + \eta_p \left[\frac{d\ln\delta}{d\ln t/to}\right]^2$$

The first two terms correspond to the terms we saw above for a fixed coordinate system, while the last two are a result of the expanding coordinates. As for the velocity, the acceleration due to coordinate expansion are negligible near $\eta = 0$ but increase with distance from the origin. The middle term is zero if $\delta \propto t$. All of the last three terms are zero if $\delta = constant$ (i.e., no expansion of coordinate system), and the result is to within the factor δ the same as equation 11.

Let's consider what happens if an oscillator operates in expanding space, $\vec{\eta} = (\vec{x} - \vec{x}_{osc})/\delta(t)$, and with logarithmic time, $\tau = \ln(t - t_{osc})/t_o$. Its location is at fixed $\vec{\eta}_{osc}$ and it begins at time t_{osc}. The phase at any time and location would be given by:

$$\phi = \vec{\kappa}_* \cdot (\vec{\eta} - \vec{\eta}_{osc}) - \omega_*(\tau - \tau_{osc}). \tag{55}$$

$\vec{k}^* = \nabla_{\vec{\eta}}\phi$ and $\omega^* = -d\phi/d\tau$ are the dimensionless wavenumber vector and frequency in our spatially scaled and logarithmic time space.

The important question for us on earth is: what do we see in our linear time, linear space coordinates? The answer can be found from our earthly definitions of 'frequency', 'wavenumber' and 'phase velocity' which given respectively as the $\omega = -d\phi/dt$, $\vec{k} = \nabla_{\vec{x}}\phi$, and $c = \omega/|\vec{k}|$.

It follows immediately that:

$$\omega = \frac{d\phi}{dt} = \frac{\omega^*}{t} - \vec{k}^* \cdot [\vec{\eta} - \vec{\eta}_{osc}] \left[\frac{1}{\delta}\frac{d\delta}{dt}\right] \tag{56}$$

$$\vec{k} = \nabla_{\vec{x}}\phi = \frac{\vec{k}^*}{\delta(t)} \tag{57}$$

Equation 56 clearly implies a red-shift for all non-negative expansion rates. And the farther away and earlier it began, the greater the shift. **In fact as noted before, it implies a red-shift even if there is no expansion at all!** (i.e., $\delta = constant$). So a red-shift alone does not imply expansion, at least if time were logarithmic.

The phase velocity seen in our earthy system would be:

$$c = \frac{\omega^*}{k^*}\left[\frac{\delta(t)}{t}\right] - |\vec{\eta} - \vec{\eta}_{osc}| \left[\frac{d\delta}{dt}\right] \tag{58}$$

If $d\delta/dt = 0$, the speed of light we see would be proportional to just the inverse age of the universe in linear time. Assuming $c^* = \omega^*/k^* = constant$, demands that c measured by us would be time dependent. But expanding $1/t$ about any time in today's epoch means any discrepancies noted would be of order $\delta t/t_p$, and as noted above well below our ability to measure it.

The linear expansion rate case is again of particular interest since $\delta = [d\delta/dt]t$, so equation 37 reduces to:

$$c = \left[\frac{d\delta}{dt}\right]\{c^* - |\vec{\eta} - \vec{\eta}_{osc}|\} \tag{59}$$

If $c = constant$ is indeed a law of physics as Einstein (and data) suggest, then this is probably enough basis to conclude that $d\delta/dt = 0$. Or that we have been wrong about the constancy of c. Alternatively we could argue the same from the assumption that $c^* = constant$ is a law of physics.

15 Summary and conclusions

In view of the above it is clear that we could have equally written Newton's law in logarithmic time without ever being able to tell the difference, at least as long as we only applied it to times within our human experience. Any differences would have shown up only for very large times – on the order of billions of years.

The most important difference for mechanics is that the definitions of mass and velocity change. In particular our traditional definition of mass becomes dependent on absolute time squared. This has important implications for the gravitational law and our applications of it. In particular what we might have previously believed to be the gravitational constant in fact varies as the fourth power of absolute time. Not recognizing this can lead to gross over-estimates of celestial masses, and incorrect inferences that mass is missing.

It is further suggested if time is indeed logarithmic, then the remaining laws of physics must be treated the same way. Maxwell's equations in particular have exactly the same form when expressed in logarithmic time. And the same is true for the important relativistic energy equations and the Minkowski invariant.

Finally, two different scenarios are considered: a first where space is not expanding, and a second where it is. The second with a constant expansion rate leads to a redshift which is linear in distance from the observer and is consistent with a constant logarithmic speed of light.

Assuming there is nothing obviously wrong with the analyses presented above, the possibility of that time might be logarithmic should be a boon for cosmologists and astronomers. It is for the former to flesh out the mathematical consequences on other aspects of our knowledge. And the latter alone have the data and wherewithal to test whether it describes their data. The late great solid mechanics experimentalist, James C. Bell (of the Johns Hopkins University), often remarked "Experimentalists test and sort theories." At very least the experimental astrophysics community has another theory to add to the mix.

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